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Quantum field Theory

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- Feynman rules:
  - draw all connected diagrams
  - vertex: take i times coefficient of corresponding operator in $\mathcal{L}$
  - propagator: take i times inverse of corresponding bilinear operator
  - external particles: take “projection” of corresponding operator

- QED Lagrangian: $\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \bar{\psi}(i\not\partial - m)\psi - e\bar{\psi}\not A\psi$

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- spinor fields: $\psi_\alpha(x) = \int \frac{d^3k}{(2\pi)^2\, 2\omega_k} \sum_s \left(b(s,k) u_\alpha(s,k) e^{-i\,kx} + d^\dagger(s,k) v_\alpha(s,k) e^{i\,kx} \right)$

  with $(k\!\!\!/ - m)u(s,k) = (k\!\!\!/ + m)v(s,k) = 0$ (Dirac)

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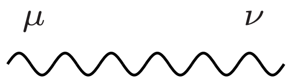
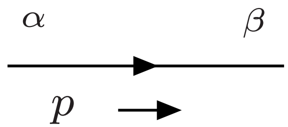
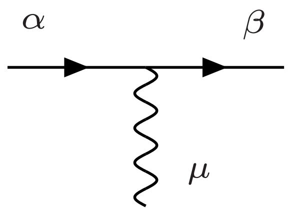
- vector fields: $A_\mu(x) = \int \frac{d^3k}{(2\pi)^3\, 2\omega_k} \sum_\lambda \left(a(\lambda,k) \epsilon_\mu(\lambda,k) e^{-i\,kx} + a^\dagger(\lambda,k) \epsilon^*_\mu(\lambda,k) e^{i\,kx} \right)$

  with $k^2 = 0$ (Maxwell) and $k\,\epsilon(\lambda,k) = 0$ (Lorentz gauge $\partial_\mu A^\mu = 0$)

for every ...	draw ...	write ...
internal photon line		$\dfrac{-ig^{\mu\nu}}{p^2+i0^+}$
internal fermion line		$\dfrac{i(\not{p}+m)_{\beta\alpha}}{p^2-m^2+i0^+}$
vertex		$-ie\gamma^{\mu}_{\beta\alpha}$
outgoing/incoming electron		$\bar{u}_\alpha(s,p)$ / $u_\alpha(s,p)$
outgoing/incoming positron		$v_\alpha(s,p)$ / $\bar{v}_\alpha(s,p)$
outgoing/incoming photon		$\epsilon^{*\mu}(\lambda,p)$ / $\epsilon^{\mu}(\lambda,p)$

- s and λ fix the polarization of the electron and photon respectively
- μ, ν are Lorentz indices, α, β are spinor indices
- conserve momentum at every vertex and integrate over all internal momenta