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Investigation of the relationship between a_μ and mass corrections in a model with vector-like leptons

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Nico Albert
geboren am 11.06.2000 in Plauen

Institut für Kern- und Teilchenphysik
Fakultät Physik
Bereich Mathematik und Naturwissenschaften
Technische Universität Dresden
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1. Gutachter: Prof. Dr. Dominik Stöckinger
2. Gutachter: Prof. Dr. Kai Zuber

Summary

Abstract
English:

In this thesis, contributions to the muon mass and the muons anomalous magnetic moment through mixing with vector-like leptons are investigated. After introducing the model, mass corrections are derived at tree- and 1-loop-level, and are compared to each other. It will be shown that the 1-loop contribution dominates in the limit of large vector-like lepton masses. In this limit, a close connection between mass corrections and corrections to the magnetic moment is established: $\delta a_\mu \sim \mathcal{O}(\delta m_\mu / m_\mu)\frac{m_\mu^2}{M_{\text{VLL}}^2}$. It will be shown that this relationship persists in the limit of large masses, even when the tree-level correction is taken into account as well. In the case of smaller masses, deviations from this relation are examined numerically.

Abstract
Deutsch

In dieser Bachelor-Arbeit werden Beiträge zur Myonmasse und zum anomalen magnetischen Moment des Myons durch Mixing mit vector-like Leptons untersucht. Das Modell wird vorgestellt, danach werden Massenkorrekturen auf tree-level und 1-Schleifen-Niveau hergeleitet und miteinander verglichen. Es wird gezeigt dass der 1-Schleifen-Beitrag im Limes großer vector-like Lepton-Massen dominiert. In diesem Limes wird eine enge Verbindung zwischen Massenkorrekturen und Korrekturen zum magnetischen Moment aufgezeigt: $\delta a_\mu \sim \mathcal{O}(\delta m_\mu / m_\mu)\frac{m_\mu^2}{M_{\text{VLL}}^2}$. Es wird gezeigt, dass dieser Zusammenhang im Fall großer Massen auch unter Berücksichtigung der tree-level-Korrektur bestehen bleibt. Im Fall kleinerer Massen wird die Abweichung von dieser Relation numerisch untersucht.

Contents

1 Introduction

Currently, the discrepancy between measurements of the muons anomalous magnetic moment $a_\mu := \frac{g-2}{2}$ and theoretical predictions by the Standard Model (SM) is one of the most promising hints on new physics beyond the Standard Model (BSM). The well celebrated recent experiment at Fermi National Laboratory (FNAL) determined this discrepancy to be [5]:

$$\delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (2.51 \pm 0.59) \cdot 10^{-9}, \tag{1.1}$$

with 4.2σ significance. There is a variety of candidates for suitable extensions of the Standard Model, such as dark photons, the two-Higgs-doublet model, Supersymmetry, and many more (for reference, see [6]). In this thesis, we will examine one possible candidate for such a model, vector-like leptons (VLL).

"Vector-like" refers to the particles being non-parity violating, meaning both left- and right-handed states behave identically under electroweak interactions. The model introduces an additional weak-isospin doublet L, which participates in electroweak interactions, and an additional singlet E, which does not. One may also consider vector-like neutrinos and quarks, but for the purpose of this thesis we will just focus on charged leptons.
Through mixing with the SM muon, VLL can contribute to physical observables. Of course, contributions to a_μ are most interesting, but we will also see that this results in sizeable modification to the muon mass. [6] state that in general, there is a simple relation between a_μ- and mass-corrections:

$$\delta a_\mu \sim \mathcal{O}\left(\frac{\delta m_\mu}{m_\mu}\right)\frac{m_\mu^2}{M^2}, \tag{1.2}$$

where M is the relevant mass scale of the respective theory.

The goal of this thesis is to examine, if and how this relation holds in the case of VLL. We will begin by introducing the overall model, as presented by [9]. We diagonalize the Yukawa-like couplings of the theory, giving rise to the muon mass correction at tree-level, which already can be comparable in size to SM Yukawa couplings.
We calculate the mass correction at 1-loop level, through self-interaction via Higgs loop, and compare it to the tree-level contribution, finding that the 1-loop contribution dominates for

sufficiently large VLL masses.
We then finally move on and discuss VLL contributions to a_μ, where we focus on the contribution from the 1-loop Higgs diagram. Studying their relations with said mass corrections, we find that in the limit of large masses (1.2) holds true, and we can derive an asymptotic expression for the proportionality factor in this case.
We finish this thesis with some numerical examinations of the general case, including the mass correction at tree-level, and discuss if the results of the asymptotic case can be generalized to smaller masses.

2 The electroweak theory of leptons

The following chapter is intended to be a brief introduction on essential theoretical concepts surrounding the description of leptons by the Standard Model of particle physics, and will mostly follow [10], [11], [12]. This overview is by no means complete, said literature provides a more sophisticated description of this complex topic.
Also, since this thesis focuses mainly on the physics of charged leptons, electroweak or strong interactions with respect to quarks will not be discussed, as well as details concerning neutrino physics.

2.1 SM leptons and electroweak unification

The Standard Model (SM) unifies electromagnetic and weak interactions, based on symmetry breaking of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_{EM}$. This model is usually referred to as the Glashow-Weinberg-Salam (GWS) model. It is important to distinguish the high-energy $\mathrm{U}(1)_Y$ from the low-energy $\mathrm{U}(1)_{EM}$: the former represents electroweak hypercharge denoted by Y, while the latter describes Quantum Electrodynamics with the usual electric charge Q. In particular, $\mathrm{U}(1)_{EM}$ does not differentiate between left- and right-handed states. $\mathrm{U}(1)_Y$ on the other hand is chiral, left- and right-handed states are assigned different hypercharges.
It is a well-known experimental fact that the weak nuclear force is maximally parity-violating, meaning only left-handed fermions participate in charged weak interactions. Using the following notation, expressing right- and left-handed states through projectors $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$

$$e_{Li} := P_L e_i$$
$$e_{Ri} := P_R e_i,$$

we represent left-handed leptons by weak-isospin doublets

$$l_{Li} = \begin{pmatrix} \nu_i \\ e_{Li} \end{pmatrix}, \ i \in \{e, \mu, \tau\} \tag{2.1}$$

which transform under SU(2)-gauge transformations. Weak isospin refers to the group charge of SU(2). We attribute $T^3 = +1/2$ for the top component of (2.1), and $T^3 = -1/2$ for the bottom one, where T^3 is the third component of the weak isospin. Right-handed leptons are

represented by weak-isospin singlets e_{Ri}, which are left invariant under SU(2)-gauge transformations. We therefore attribute $T^3 = 0$ to weak-isospin singlets. Although both singlets and doublets transform under the $\mathrm{U}(1)_Y$ subgroup; we assign $Y_L = -1/2$ for the left-handed doublet and $Y_R = -1$ for the right-handed singlet.
Weak isospin and hypercharge are related to the known electric charge by:

$$Q = T^3 + Y. \tag{2.2}$$

We will now try to write down an explicit Lagrangian for the theory:
We label the gauge fields corresponding to $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ as

$$A^1_\mu, A^2_\mu, A^3_\mu \text{ for } \mathrm{SU}(2)_L$$
$$B_\mu \text{ for } \mathrm{U}(1)_Y,$$

and separate the kinetic Lagrangian like

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{leptons}}. \tag{2.3}$$

$\mathcal{L}_{\text{gauge}}$ contains the corresponding field strength tensors for all four gauge fields. The matter part is given by Dirac-like kinetic terms for fermions with the respective covariant derivatives:

$$\mathcal{L}_{\text{leptons}} = \bar{e}_{Ri} i\gamma^\mu \left(\partial_\mu + ig'\frac{Y}{2}B_\mu\right) e_{Ri} + \bar{l}_{Li} i\gamma^\mu \left(\partial_\mu + ig'\frac{Y}{2}B_\mu + ig\frac{\vec{\tau}}{2}\cdot\vec{A}_\mu\right) l_{Li}. \tag{2.4}$$

g and g' denote the couplings of $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ respectively, $\vec{\tau} := (\tau^1, \tau^2, \tau^3)$ and Y denote the group generators. Summation over i is implied.

To complete this model, we should now introduce mass terms $m\bar{\psi}\psi$ for fermions. But, because

$$\bar{\psi}\psi = \frac{1}{2}\bar{\psi}(1-\gamma_5)\psi + \frac{1}{2}\bar{\psi}(1+\gamma_5)\psi = \bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R, \tag{2.5}$$

such terms always link left- and right-handed spinors together. Since only left-handed doublets transform under the $\mathrm{SU}(2)_L$ subgroup, those mixed terms are not gauge invariant under the theory, making them invalid.
Also, we have worked so far with four massless gauge fields $\vec{A}_\mu, B_\mu$. But in nature, only one massless gauge boson is observed, the photon.
Both problems can be fixed by introducing an additional doublet of complex scalar fields, the Higgs-doublet, which produces valid mass terms for fermions and gauge bosons through spontaneous symmetry breaking, and introduces an additional observable field - the Higgs field.

2.2 Spontaneous symmetry breaking and the Higgs mechanism

To describe the Higgs mechanism, we will first introduce the explicit potential for symmetry breaking and discuss few aspects of spontaneous symmetry breaking (ssb), before applying it to the GWS model.
Spontaneous symmetry breaking refers to the breaking of some ground state symmetry a system obeys, without breaking the symmetry present in the action, respectively the Lagrangian, of the system. In our example, this enables us to encode the breaking of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_{EM}$ without restricting the overall gauge invariance of the theory.
We introduce the Higgs doublet

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \tag{2.6}$$

and the following Lagrangian for the Higgs sector:

$$\mathcal{L}_{\text{Higgs}} = (D^\mu H)^\dagger (D_\mu H) - V(H^\dagger H). \tag{2.7}$$

D_μ denotes the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ covariant derivative, which already appeared in (2.4):

$$D_\mu = \partial_\mu + ig'\frac{Y}{2}B_\mu + ig\frac{\vec{\tau}}{2} \cdot \vec{A}_\mu. \tag{2.8}$$

For the potential, we choose

$$V(H^\dagger H) = \mu^2 (H^\dagger H) + \lambda (H^\dagger H)^2. \tag{2.9}$$

When $\mu^2 \geq 0$, this potential shows a unique global minimum: $H^\dagger H = 0 \iff H = (0,0)^\top$, which clearly is gauge invariant.
This changes if $\mu^2 < 0$: for every phase $\varphi \in [0, 2\pi)$,

$$\langle H \rangle_0 := e^{i\varphi} \begin{pmatrix} 0 \\ v \end{pmatrix}, \; v = \sqrt{\frac{-\mu^2}{2\lambda}} \tag{2.10}$$

minimizes the potential, where v is the vacuum expectation value of the Higgs field. Note that this expression is no longer gauge invariant. Without loss of generality, we can assume $\langle H \rangle_0$ to be real, equivalent to $\varphi = 0$. Experimentally, it was found that $v = \sqrt{2}M_h = \sqrt{2} \cdot (125.25 \pm 0.17)$ GeV [1].

We can now parameterize H in the following way:

$$H = \begin{pmatrix} 0 \\ v \end{pmatrix} + \begin{pmatrix} G^+ \\ \frac{h}{\sqrt{2}} + iG^0 \end{pmatrix}. \tag{2.11}$$

We call h the Higgs field, and $G^{+,0}$ Goldstone fields. Demanding, that observable physics should still be unchanged under gauge transformations, we can perform such a transformation such that G^+, G^0 are mapped to zero, and remain with:

$$H = \begin{pmatrix} 0 \\ v + \frac{h}{\sqrt{2}} \end{pmatrix}. \tag{2.12}$$

Substituting this into (2.7), we can separate terms proportional to v from those containing h. One can then perform a linear transformation

$$(A^1_\mu, A^2_\mu, A^3_\mu, B_\mu) \mapsto (W_\mu, W^\dagger_\mu, Z_\mu, A_\mu) =: (W^+_\mu, W^-_\mu, Z_\mu, A_\mu), \tag{2.13}$$

bringing the resulting mass terms into diagonal form

$$\text{(mass terms)} = M_W^2 W^\dagger_\mu W^\mu + \frac{1}{2} M_Z^2 Z_\mu Z^\mu. \tag{2.14}$$

These correspond to the tree-level masses of the observable photon, $W^\pm$- and Z^0- bosons. Immediate conclusions are that the photon has zero mass, and that $M_{W^+} = M_{W^-}$ - both in agreement with experimental observations. The h-terms describe interactions with the Higgs boson, which also contribute to the boson masses at higher order of perturbation.

We now want to study interactions of leptons with the Higgs field, so-called Yukawa interactions, which yield the requested gauge invariant mass terms for leptons.

2.3 SM Yukawa couplings and lepton masses

We will now apply the mechanism presented above, which gave rise to mass terms for the electroweak gauge bosons, to the leptons of the Standard Model.
Using that, under an arbitrary gauge transformation

$$\psi \mapsto \exp\left(i\alpha_j \sigma^j\right) \psi$$

the Dirac-adjoint $\bar{\psi}$ transforms like

$$\bar{\psi} \mapsto \bar{\psi} \exp\left(-i\alpha_j \sigma^j\right),$$

we can write down the following interaction terms:

$$\mathcal{L}_{\text{Yukawa}} = -\bar{l}_{Li} y_{ij} e_{Rj} H - H^\dagger \bar{e}_{Rj} y_{ij} l_{Li} \tag{2.15}$$

(summation over flavour indices i, j is implied). If we choose the right hypercharge for H, $Y_H = +1/2$, these terms become gauge invariant. y_{ij} are called Yukawa couplings, which we can assume to be diagonal in the case of leptons.
After ssb, we can again separate terms containing v, and obtain the requested mass terms:

$$(v\text{-terms}) = -(y_{ij} v) \bar{e}_{Li} e_{Rj} + h.c., \tag{2.16}$$

identifying the tree-level lepton masses as:

$$m_i = y_{ii} v. \tag{2.17}$$

Terms containing the Higgs field h again represent interactions with the Higgs boson, contributing to lepton masses at higher order of perturbation:

$$(h\text{-terms}) = -\bar{e}_{Li} y_{ij} e_{Rj} \frac{h}{\sqrt{2}} + h.c. \tag{2.18}$$

For reference, current experimental values for SM lepton masses are [2] [3] [4]:

$$m_e = (0.5109989461 \pm 0.0000000031)\ \text{MeV}, \tag{2.19}$$

$$m_\mu = (105.6583745 \pm 0.0000024)\ \text{MeV}, \tag{2.20}$$

$$m_\tau = (1776.86 \pm 0.12)\ \text{MeV}. \tag{2.21}$$

3 Introducing vector-like leptons

We will now move beyond the Standard Model, introducing so-called vector-like leptons (VLL) to study their effects on observable quantities like the muon mass or the muon anomalous magnetic moment through mixing with the SM muon. The term "vector-like" refers to both left- and right-handed states behaving identically under electroweak interactions - in contrast to SM fermions.
First the overall model for charged vector-like leptons, as presented by [9], will be established. Following [9], we will apply the limit

$$y_\mu v \ll \lambda_E v, \lambda_L v, \lambda v, \bar{\lambda} v \ll M_E, M_L. \tag{3.1}$$

We will discuss couplings between vector-like leptons and the SM Higgs boson, before moving on to said observables in following chapters.
Vector-like quarks and neutrinos are also well established candidates for BSM physics, those will not be discussed in this thesis.

3.1 The model

We introduce two additional vector-like doublets:

$$L_{L,R} = \begin{pmatrix} L^0_{L,R} \\ L^-_{L,R} \end{pmatrix} \tag{3.2}$$

as well as two additional vector-like singlets $E_{L,R}$. The quantum numbers of the VLL, compared to the relevant SM particles, are displayed in table 3.1. E_R and L_L have the same quantum numbers as e_R and l_L, respectively. We can apply the same structure as in (2.15) to combine

	l_{Li}	e_{Ri}	H	$L_{L,R}$	$E_{L,R}$
$\mathrm{SU}(2)_L$	**2**	**1**	**2**	**2**	**1**
$\mathrm{U}(1)_Y$	$-\frac{1}{2}$	-1	$\frac{1}{2}$	$-\frac{1}{2}$	-1

Table 3.1: Quantum numbers of relevant SM and VLL particles. Every doublet (**2**) is assgined $T^3 = +1/2$ for the top component and $T^3 = -1/2$ for the bottom component, singlets (**1**) carry $T^3 = 0$. The second row shows the respective hypercharges Y. The electric charge is given by $Q = T^3 + Y$.

vector-like singlets and doublets with each other and with SM leptons, to obtain additional Yukawa-like interactions terms. Since left- and right-handed VLL transform identical under $\mathrm{SU}(2)_L$, we are also allowed to add explicit mass terms $M_L \bar{L}_L L_R$ and $M_E \bar{E}_L E_R$. We obtain:

$$\begin{aligned} \mathcal{L}_{\text{mass}}^{\text{VLL}} = &- \bar{l}_{Li} y_{ij} e_{Rj} H - \bar{l}_{Li} \lambda_{Ei} E_R H - \bar{L}_L \lambda_{Lj} e_{Rj} H - \lambda \bar{L}_L E_R H - \bar{\lambda} H^\dagger \bar{E}_L L_R \\ &- M_L \bar{L}_L L_R - M_E \bar{E}_L E_R + h.c. \end{aligned} \tag{3.3}$$

This is the most general renormalizable Lagrangian. λ_{Ei} and λ_{Li} couple VLL to standard model leptons, λ and $\bar{\lambda}$ couple VLL to each other. We assume all those couplings to be real-valued, which corresponds to the theory preserving CP-symmetry.
These Yukawa-like mass and interaction terms obey now a more complicated, non-diagonal structure than in the case of SM leptons. To gain more insight one can perform a basis transformation, such that all mass terms become diagonal. We will precede in performing this diagonalization hereinafter.

3.2 Mass eigenstates and diagonalization

After ssb, we can express the emerging mass terms of the Lagrangian (3.3) in matrix form:

$$(\bar{e}_{Li}, \bar{L}_L^-, \bar{E}_L) M_e \begin{pmatrix} e_{Rj} \\ L_R^- \\ E_R \end{pmatrix} = (\bar{e}_{Li}, \bar{L}_L^-, \bar{E}_L) \begin{pmatrix} y_{ij} v & 0 & \lambda_{Ei} v \\ \lambda_{Lj} v & M_L & \lambda v \\ 0 & \bar{\lambda} v & M_E \end{pmatrix} \begin{pmatrix} e_{Rj} \\ L_R^- \\ E_R \end{pmatrix}, \tag{3.4}$$

where M_e is a 5×5 matrix. We simplify notation through introducing the 5-component vectors $e_{La} := (e_{Li}, L_L^-, E_L)^\top$ and $e_{Ra} := (e_{Ri}, L_R^-, E_R)^\top$, where $a, b, c, ...$ index the combined vectors and $i, j, k, ...$ index SM leptons. Assuming the SM Yukawa couplings are diagonal, we can set $\lambda_{E1} = \lambda_{E3} = 0 = \lambda_{L1} = \lambda_{L3}$ and define $\lambda_E := \lambda_{E2}$, $\lambda_L := \lambda_{L2}$, since we are only interested in VLL mixing with the muon. SM couplings of electron and tau are not modified, their respective masses fully originate from SM relations. Therefore, it is sufficient to just focus on the 3×3 matrix

$$M_e = \begin{pmatrix} y_\mu v & 0 & \lambda_E v \\ \lambda_L v & M_L & \lambda v \\ 0 & \bar{\lambda} v & M_E \end{pmatrix}, \tag{3.5}$$

where $y_\mu := y_{22}$.
M_e is a well behaved matrix, so there exists a bi-unitary transformation

$$(e_{La}) := \begin{pmatrix} e_{L\mu} \\ L_L^- \\ E_L \end{pmatrix} \mapsto U_L \begin{pmatrix} e_{L\mu} \\ L_L^- \\ E_L \end{pmatrix} =: (\hat{e}_{La}) \tag{3.6}$$

(similarly for $L \to R$), such that $U_L^\dagger M_E U_R$ becomes diagonal, defining the mass-eigenstate basis $(\hat{e}_{La})$. We label the eigenvalues as follows:

$$U_L^\dagger \begin{pmatrix} y_\mu v & 0 & \lambda_E v \\ \lambda_L v & M_L & \lambda v \\ 0 & \bar{\lambda} v & M_E \end{pmatrix} U_R = \begin{pmatrix} m_\mu & 0 & 0 \\ 0 & m_{e_4} & 0 \\ 0 & 0 & m_{e_5} \end{pmatrix}. \tag{3.7}$$

The two diagonalization matrices are given by [9]:

$$U_L = \begin{pmatrix} 1 - v^2 \frac{\lambda_E^2}{2M_E^2} & -v^2 \left(\frac{\lambda_E}{M_L} \frac{\bar{\lambda} M_E + \lambda M_L}{M_E^2 - M_L^2} - \frac{y_\mu \lambda_L}{M_L^2} \right) & v \frac{\lambda_E}{M_E} \\ v^2 \frac{\bar{\lambda} \lambda_E M_L - y_\mu \lambda_L M_E}{M_L^2 M_E} & 1 - v^2 \frac{(\lambda M_E + \bar{\lambda} M_L)^2}{2(M_E^2 - M_L^2)^2} & v \frac{\bar{\lambda} M_L + \lambda M_E}{M_E^2 - M_L^2} \\ -v \frac{\lambda_E}{M_E} & -v \frac{\bar{\lambda} M_L + \lambda M_E}{M_E^2 - M_L^2} & 1 - v^2 \frac{\lambda_E^2}{2M_E^2} - v^2 \frac{(\lambda M_E + \bar{\lambda} M_L)^2}{2(M_E^2 - M_L^2)^2} \end{pmatrix} \tag{3.8}$$

$$U_R = \begin{pmatrix} 1 - v^2 \frac{\lambda_L^2}{2M_L^2} & v \frac{\lambda_L}{M_L} & v^2 \left(\frac{\lambda_L}{M_E} \frac{\bar{\lambda} M_L + \lambda M_E}{M_E^2 - M_L^2} + \frac{y_\mu \lambda_E}{M_E^2} \right) \\ -v \frac{\lambda_L}{M_L} & 1 - v^2 \frac{\lambda_L^2}{2M_L^2} - v^2 \frac{(\bar{\lambda} M_E + \lambda M_L)^2}{2(M_E^2 - M_L^2)^2} & v \frac{\bar{\lambda} M_E + \lambda M_L}{M_E^2 - M_L^2} \\ v^2 \frac{\lambda_L \bar{\lambda} M_E - y_\mu \lambda_E M_L}{M_L M_E^2} & -v \frac{\bar{\lambda} M_E + \lambda M_L}{M_E^2 - M_L^2} & 1 - v^2 \frac{(\bar{\lambda} M_E + \lambda M_L)^2}{2(M_E^2 - M_L^2)^2} \end{pmatrix}. \tag{3.9}$$

Applying limit (3.1), we find no significant corrections to VLL masses M_E, M_L, so we can assume $m_{e_4} \simeq M_L, m_{e_5} \simeq M_E$. This is not the case for the muon mass m_μ, those corrections will be discussed in section 4.1.

To move beyond tree-level, and calculate corrections to the anomalous magnetic moment or higher order mass corrections, we need to study interactions between VLL, muon and Higgs boson. We begin by studying and calculating the necessary Higgs coupling constants in the next section.

3.3 Higgs couplings

In our scenario, couplings of electron and tau are not modified and are identical to the respective SM couplings: $\lambda_{e,\tau} = m_{e,\tau}/v$. Because we added explicit vector-like mass terms, this relation does not hold for muon, L and E.
Interaction terms with the Higgs boson are of similar form as (2.18). Applying this to Yukawa-

like terms of (3.3), we obtain Lagrangian:

$$\mathcal{L}_{\text{int}}^{\text{VLL}} = -\frac{1}{\sqrt{2}}\bar{e}_{La}Y_{ab}e_{Rb}h + h.c. = -\frac{1}{\sqrt{2}}\bar{\hat{e}}_{La}(U_L^\dagger)_{ac}Y_{cd}(U_R)_{db}\hat{e}_{Rb}h + h.c. \tag{3.10}$$

(again, summation over a, b, c, d is implied). The matrix Y is given by:

$$Y = \begin{pmatrix} y_\mu & 0 & \lambda_E \\ \lambda_L & 0 & \lambda \\ 0 & \bar{\lambda} & 0 \end{pmatrix} = \frac{1}{v}\left(M_e - \text{diag}(0, M_L, M_E)\right). \tag{3.11}$$

This leaves us with the following expression for Higgs coupling λ_{ab}:

$$\lambda_{ab} = \sum_{c,d=2,4,5} (U_L^\dagger)_{ac}Y_{cd}(U_R)_{db}, \tag{3.12}$$

or equivalently:

$$(\lambda_{ab}v) = \begin{pmatrix} m_\mu & 0 & 0 \\ 0 & m_{e_4} & 0 \\ 0 & 0 & m_{e_5} \end{pmatrix} - U_L^\dagger \begin{pmatrix} 0 & 0 & 0 \\ 0 & M_L & 0 \\ 0 & 0 & M_E \end{pmatrix} U_R. \tag{3.13}$$

Expression (3.13) explicitly shows the deviation from usual SM-relations for Higgs couplings, as a result of explicit vector-like mass terms.

In limit (3.1), we calculate couplings relevant for later discussions, up to leading order in $\frac{\lambda v}{M_{E,L}}$ (where λ is shorthand for $\lambda, \bar{\lambda}, \lambda_{E,L}$):

$$\begin{aligned} \lambda_{\mu 4} &= -\lambda_E \frac{\bar{\lambda} v M_E + \lambda v M_L}{M_E^2 - M_L^2} - \bar{\lambda}\lambda_E \frac{v}{M_E} \\ \lambda_{4\mu} &= \lambda_L \\ \lambda_{\mu 5} &= \lambda_E \\ \lambda_{5\mu} &= \lambda_L \frac{\lambda v M_E + \bar{\lambda} v M_L}{M_E^2 - M_L^2} - \bar{\lambda}\lambda_L \frac{v}{M_L}. \end{aligned} \tag{3.14}$$

An immediate question that comes to mind is: What happens in the case of $M_E = M_L$ - the couplings seem to diverge. This technical mathematical problem should resolve itself when carrying out diagonalization (3.7) again for this specific case. We will later see that those divergencies cancel with each other when calculating physical observables, although it is questionable if those results are accurate for $M_E = M_L$. However, from a physical perspective $M_E = M_L$ is no more interesting than any other set of values (within a reasonable range), so we will ignore this technical problem.

4 Corrections to the muon mass

Using the tools presented in earlier sections, we are now able to discuss the effects of VLL-mixing on interesting physical observables. In this chapter, we will examine modifications of the muon mass, before moving on to the muons magnetic moment in the following one.
We will begin by calculating contributions at tree-level, which were already studied by [9]. After that, we move on to higher order of perturbation and derive an explicit formula for the muon self-energy through interaction with the Higgs boson. From there we can directly calculate the desired 1-Loop mass correction. We finish this chapter with a short comparison of both contributions, where we are especially interested in their behaviour in the limit of large VLL masses.

4.1 Tree-level corrections

The muon mass at tree-level is given by the emerging mass terms after ssb. In the mass-eigenstate basis those become diagonal, and we can directly read off the contributions from the eigenvalues of (3.7). Applying limit (3.1), we find that only one term gives contributions comparable in size to the SM Yukawa coupling y_μ, in agreement with [9]. The remaining terms are neglected. The result is:

$$\begin{aligned} m_\mu^{\text{tree-level}} &= (U_L^\dagger M_e U_R)_{11} = y_\mu v + \lambda_L \bar{\lambda} \lambda_E \frac{v}{M_L} \frac{v}{M_E} v + (\text{small terms}) \\ &=: m_\mu^H + m_\mu^{LE} + (\text{small terms}). \end{aligned} \tag{4.1}$$

m_μ^H originates from the usual SM Yukawa couplings, m_μ^{LE} denotes the dominant contribution through mixing with VLL. Tracing back the mass terms to before ssb, we can use this result to formulate an effective Lagrangian for the muon mass at tree-level:

$$\mathcal{L}_{\text{eff}}^\mu = -\bar{\mu}_L \left(y_\mu + \frac{\lambda_L \bar{\lambda} \lambda_E}{M_L M_E} H H^\dagger \right) \mu_R H + h.c. \tag{4.2}$$

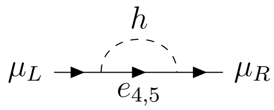
4.2 1-Loop corrections

To obtain higher order mass corrections, we first need to calculate the muon self-energy through Higgs-interaction, $\Sigma(p)$. The mass correction is then given by:

$$m_\mu^{1-\text{Loop}} = -\Sigma(p = m_\mu). \tag{4.3}$$

The sign on the right hand side depends on the underlying renormalization scheme, the convention used above follows the on-shell scheme.
The muon self-energy is given by the amplitude of the following 1-Loop diagram:

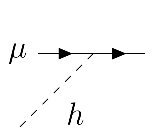
$$i\Sigma(p) = \tag{4.4}$$



The relevant interaction Lagrangian (3.10), after carrying out the diagonalization, becomes:

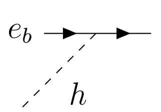
$$\begin{aligned}\mathcal{L}_{\text{int}}^{\text{VLL}} &= -\frac{1}{\sqrt{2}}\bar{e}_{La}\lambda_{ab}e_{Rb}h - \frac{1}{\sqrt{2}}h^\dagger\bar{e}_{Rb}\lambda_{ab}e_{La} \\ &= -\frac{1}{\sqrt{2}}\bar{e}_a(\lambda_{ab}P_R)e_bh - \frac{1}{\sqrt{2}}h^\dagger\bar{e}_b(\lambda_{ab}^*P_L)e_a.\end{aligned} \tag{4.5}$$

Note that since we assume all couplings to be real-valued, the differentiation between λ_{ab} and λ_{ab}^* is not necessary. However, calculations become only slightly more extensive when respecting this, so we will stick to the general case for now. Rewriting (4.5) in terms of $P_{L,R}$ enables us to directly read off the respective vertex factors:



$$= -\frac{i}{\sqrt{2}}(\lambda_{2b}P_L + \lambda_{b2}P_R) =: i(c_bP_L + d_bP_R) \tag{4.6}$$

and



$$= -\frac{i}{\sqrt{2}}(\lambda_{b2}^*P_L + \lambda_{2b}^*P_R) =: i(d_b^*P_L + c_b^*P_R),$$

where $b \in \{4, 5\}$.
We denote the muon momentum by p, the VLL momentum inside the loop by q, and the Higgs momentum by $(p - q)$. Applying the respective Feynman rules, the amplitude for the

amputated diagram becomes:

$$\begin{aligned} i\Sigma(p) &= \sum_{b=4,5} \int \frac{d^4q}{(2\pi)^4} i(d_b^* P_L + c_b^* P_R) \frac{i}{\not{q} - m_b} i(c_b P_L + d_b P_R) \frac{i}{(p-q)^2 - M_h^2} \\ &= i^4 \sum_{b=4,5} \int \frac{d^4q}{(2\pi)^4} \frac{(d_b^* P_L + c_b^* P_R)(\not{q} + m_b)(c_b P_L + d_b P_R)}{[q^2 - m_b^2][(p-q)^2 - M_h^2]}. \end{aligned} \tag{4.7}$$

We take a closer look at the numerator: Applying the identity

$$\gamma^\mu P_L = P_R \gamma^\mu \text{ and } \gamma^\mu P_R = P_L \gamma^\mu, \tag{4.8}$$

we can interchange $\not{q}$ and $P_{L,R}$ and use properties of the projectors

$$P_{L,R}^2 = P_{L,R} \text{ and } P_L P_R = P_R P_L = 0 \tag{4.9}$$

to simplify the expression further, and obtain:

$$(\text{numerator}) = m_b(c_b^* d_b P_R + d_b^* c_b P_L) + \not{q}(|c_b|^2 P_L + |d_b|^2 P_R). \tag{4.10}$$

When acting on external spinors, the projectors $P_{L,R}$ just contribute an overall factor of $\frac{1}{2}$ to the integral. With this, intergral (4.7) becomes:

$$i\Sigma(p) = \sum_{b=4,5} \int \frac{d^4q}{(2\pi)^4} \frac{m_b \operatorname{Re}(c_b^* d_b) + \frac{1}{2}\not{q}(|c_b|^2 + |d_b|^2)}{[q^2 - m_b^2][(p-q)^2 - M_h^2]}. \tag{4.11}$$

These integrals can now be solved with standard methods. The result can be expressed in terms of so-called loop functions B_0, B_1:

$$\begin{aligned} i\Sigma(p) &= \sum_{b=4,5} \left[m_b \operatorname{Re}(c_b^* d_b) \frac{i}{16\pi^2} B_0(p, m_b, M_h) - \frac{1}{2}\not{p}(|c_b|^2 + |d_b|^2) \frac{i}{16\pi^2} B_1(p, m_b, M_h) \right] \\ &= \frac{i}{32\pi^2} \sum_{b=4,5} \left[m_b \operatorname{Re}(\lambda_{2b}^* \lambda_{b2}) B_0(p, m_b, M_h) - \frac{1}{2}\not{p}(|\lambda_{2b}|^2 + |\lambda_{b2}|^2) B_1(p, m_b, M_h) \right]. \end{aligned} \tag{4.12}$$

Dealing with these loop functions can be quite complicated and unpractical, we simplify our considerations by restricting ourselves to the limit $p \to 0$. Because we are only interested in $\Sigma(p = m_\mu)$, and $m_\mu \ll m_b, M_h$, this is justified.

For $p = 0$, $B_0(0, m_b, M_h)$ can be expressed through the following exact formula:

$$B_0(0, m_b, M_h) = \frac{A_0(m_b) - A_0(M_h)}{m_b^2 - M_h^2}, \text{ where} \tag{4.13}$$

$$A_0(m) = m^2 \left(\ln \frac{\mu^2}{m^2} + 1 \right). \tag{4.14}$$

μ is some renormalization scale, we choose $\mu = m_b$.
We can also approximate B_1 via:

$$B_1(p, m_b, M_h) \sim -\frac{1}{2} B_0(p, m_b, M_h) \ (p \to 0). \tag{4.15}$$

We simplify notation by:

$$B_0(0, m_b, M_h) = 1 - \frac{\ln \frac{m_b^2}{M_h^2}}{\frac{m_b^2}{M_h^2} - 1} =: \tilde{B}_0 \left(\frac{m_b^2}{M_h^2} \right) \tag{4.16}$$

and

$$B_1(0, m_b, M_h) \simeq -\frac{1}{2} \tilde{B}_0 \left(\frac{m_b^2}{M_h^2} \right). \tag{4.17}$$

Applying this to (4.12), $\Sigma(p)$ is given, in very good approximation, by:

$$i\Sigma(p) = \frac{i}{32\pi^2} \sum_{b=4,5} \left[m_b \operatorname{Re}(\lambda_{2b}^* \lambda_{b2}) + \frac{1}{4} \not{p} (|\lambda_{2b}|^2 + |\lambda_{b2}|^2) \right] \tilde{B}_0 \left(\frac{m_b^2}{M_h^2} \right). \tag{4.18}$$

And finally:

$$\begin{aligned} m_\mu^{\text{1-Loop}} &= -\Sigma(p = m_\mu) \\ &= -\frac{1}{32\pi^2} \sum_{b=4,5} \left[m_b \operatorname{Re}(\lambda_{2b}^* \lambda_{b2}) + \frac{m_\mu}{4} (|\lambda_{2b}|^2 + |\lambda_{b2}|^2) \right] \tilde{B}_0 \left(\frac{m_b^2}{M_h^2} \right). \end{aligned} \tag{4.19}$$

We can simplify a little bit further: Since $m_\mu \ll m_{4,5}$, the second summand in [...] adds negligible contributions compared to the first one. We obtain the following compact formula for the mass correction at 1-Loop level:

$$m_\mu^{\text{1-Loop}} \simeq -\frac{1}{32\pi^2} \sum_{b=4,5} m_b \operatorname{Re}(\lambda_{2b}^* \lambda_{b2}) \tilde{B}_0 \left(\frac{m_b^2}{M_h^2} \right). \tag{4.20}$$

4.3 Comparing m_μ^{LE} vs. $m_\mu^{\text{1-Loop}}$

In this last section we want to discuss the significance of both tree-level and 1-loop-level mass corrections. Detailed calculations of $m_\mu^{\text{1-Loop}}$ in the limit of large masses will be shown in section 5.2 - at this point we will just present the results. Following estimates are based on equation (5.19).
Roughly speaking, because $m_\mu^{\text{1-Loop}} \sim \textit{const}$ and $m_\mu^{LE} \sim \frac{1}{M_{L,E}^2}$ for large masses $M_{L,E}$, we expect

the 1-loop correction to dominate in this limit. We estimate

$$\frac{m_\mu^{\text{1-Loop}}}{m_\mu^{LE}} \simeq -\frac{1}{32\pi^2}\frac{M^2}{v^2}\left(\frac{\lambda}{\bar{\lambda}} - \frac{c^2+1}{c}\right), \tag{4.21}$$

where $M := M_L$ defines a single mass scale and $M_E =: cM$. Solving $\frac{m_\mu^{\text{1-Loop}}}{m_\mu^{LE}} = 1$ for M roughly yields the threshold from which the 1-loop correction dominates, for $\lambda/\bar{\lambda} \simeq 1$ and $c \simeq 1$ we obtain $M \simeq \sqrt{32}\pi v \simeq 3$ TeV.

Following figures (4.1) - (4.3) show that this pretty rough estimate is quite accurate. They display $m_\mu^{\text{1-Loop}}/m_\mu$ and m_μ^{LE}/m_μ with varying M in the same plot, respectively. $\bar{\lambda}$ is fixed to $\bar{\lambda} = 0.5$, as well as $\lambda_{L,E}$ according to electroweak precision data [7] [9] (see section 5.3 for more details). The figures also very clearly show the λ - dependence of the 1-loop mass correction, in contrast to the tree-level correction.

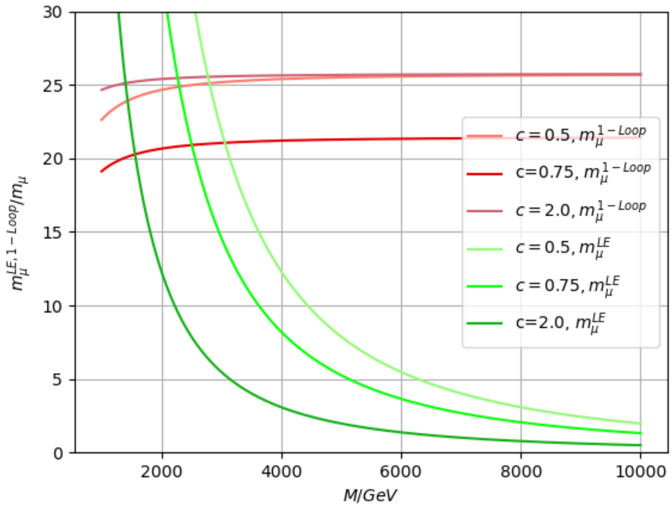


Figure 4.1: $m_\mu^{\text{1-Loop}}/m_\mu$ and m_μ^{LE}/m_μ for different values of c, with fixed $\lambda = 0, \bar{\lambda} = 0.5$

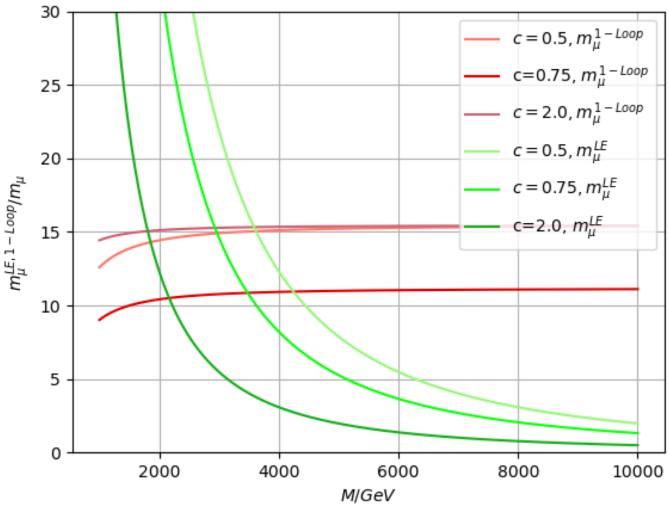


Figure 4.2: $m_\mu^{\text{1-Loop}}/m_\mu$ and m_μ^{LE}/m_μ for different values of c, with fixed $\lambda = 0.5, \bar{\lambda} = 0.5$

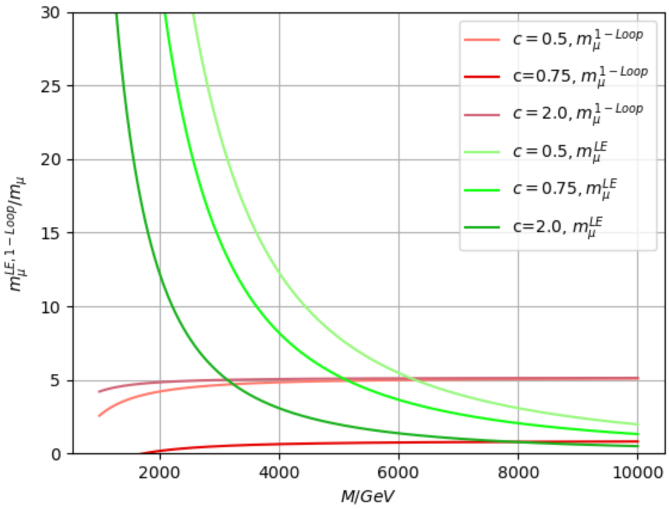


Figure 4.3: $m_\mu^{\text{1-Loop}}/m_\mu$ and m_μ^{LE}/m_μ for different values of c, with fixed $\lambda = 1.0, \bar{\lambda} = 0.5$

5 Discussion of the muon anomalous magnetic moment

In this last chapter, we will discuss VLL contributions to the anomalous magnetic moment of the muon, and study their relations with the already discussed mass corrections.
Through general considerations, it was shown by [6] that the following parameterization can be established:

$$\delta a_\mu^{\mathrm{BSM}} = C_{\mathrm{BSM}} \frac{m_\mu^2}{M_{\mathrm{BSM}}^2}, \tag{5.1}$$

where M_{BSM} is the relevant mass scale of the theory and the coefficient C_{BSM} depends on model details like couplings, electroweak vacuum expectation values and more. BSM models that modify a_μ typically also modify the muon mass, we already saw this in the case of VLL in the preceding chapter. [6] state that those modifications usually scale like

$$\frac{\delta m_\mu^{\mathrm{BSM}}}{m_\mu} \sim \mathcal{O}(C_{\mathrm{BSM}}), \tag{5.2}$$

so we can estimate $\delta a_\mu^{\mathrm{BSM}} \sim \mathcal{O}(\delta m_\mu^{\mathrm{BSM}}/m_\mu)\frac{m_\mu^2}{M^2}$. However, as [9] was able to show, this does not hold for VLL if only tree-level mass corrections are considered:

$$\delta a_\mu^{\mathrm{VLL}} : \frac{m_\mu^{LE}}{m_\mu} \sim \frac{m_\mu^2}{16\pi^2 v^2}. \tag{5.3}$$

The overall goal of this chapter is to show that (5.2) does indeed hold when mass corrections at 1-loop level are taken into account. As shown in section 4.3, those 1-loop corrections dominate in the limit of large VLL masses, $M_E^2, M_L^2 \gg M_h^2$. We can perform analytical studies in this limit, neglecting the tree-level correction, and show that (5.2) holds true in this case. After that, we will perform numerical studies and check, whether or not the analytical results can be generalized.

We start with some preliminaries, shortly discussing the formula for $\delta a_\mu^{\mathrm{VLL}}$ already derived by [9] in the following section.

5.1 Preliminaries

Figure (5.1) shows the relevant diagrams for contributions to a_μ through extra fermions [9]. They are given by loops involving the Higgs boson, W- and Z-bosons and VLL.

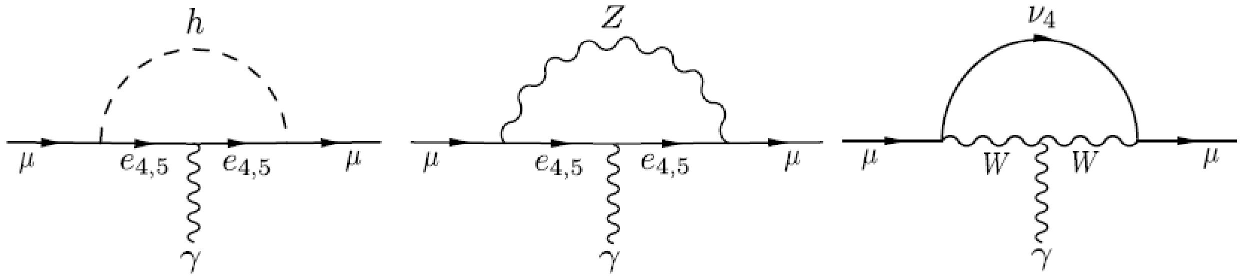


Figure 5.1: Feynman diagrams of contributions to a_μ containing VLL at 1-loop level

In this thesis, we will focus only on the contribution originating from the Higgs loop. The other two contributions are similar in structure, and are discussed in detail in [9]. But they are by no means negligible, because of that we cannot make any meaningful predictions on specific values of free parameters. Instead we focus on the overall behaviour of a_μ and its relation to mass corrections originating from VLL.
The contribution to a_μ through the Higgs diagram is given by [9]:

$$\delta a_\mu^h = -\frac{m_\mu}{32\pi^2 M_h^2} \sum_{b=4,5} \left[m_b \,\mathrm{Re}(\lambda_{2b}\lambda_{b2}) G_h\left(\frac{m_b^2}{M_h^2}\right) + m_\mu (|\lambda_{2b}|^2 + |\lambda_{b2}|^2) F_h\left(\frac{m_b^2}{M_h^2}\right)\right], \tag{5.4}$$

with loop functions:

$$F_h(x) = -\frac{x^3 - 6x^2 + 3x + 6x\ln(x) + 2}{6(1-x)^4} \tag{5.5}$$

$$G_h(x) = \frac{x^2 - 4x + 2\ln(x) + 3}{(1-x)^3}. \tag{5.6}$$

As in (4.20), we can neglect summands $\propto m_\mu$. This is justified, because $F_h(m_b^2/M_h^2)$ and $G_h(m_b^2/M_h^2)$ are of roughly the same order of magnitude in the range of interesting masses. We obtain:

$$\delta a_\mu^h \simeq -\frac{m_\mu}{32\pi^2 M_h^2} \sum_{b=4,5} m_b \,\mathrm{Re}(\lambda_{2b}\lambda_{b2}) G_h\left(\frac{m_b^2}{M_h^2}\right). \tag{5.7}$$

Note that this expression is already very similar in structure compared to (4.20), which is very promising.

5.2 Analytical studies in the limit of large VLL masses

In the following we want to show that

$$\delta a_\mu : \frac{m_\mu^{\text{1-Loop}}}{m_\mu} \sim C_{\text{VLL}} \frac{m_\mu^2}{M_{E,L}^2} \ (M_{E,L} \to \infty), \tag{5.8}$$

and determine an asymptotic expression for C_{VLL} in this limit.
Combining (4.20) and (5.7), we find the following general expression for the left hand side:

$$\delta a_\mu : \frac{m_\mu^{\text{1-Loop}}}{m_\mu} = \frac{m_\mu^2}{M_h^2} \frac{\sum_{b=4,5} m_b \operatorname{Re}(\lambda_{2b}\lambda_{b2}) G_h\left(\frac{m_b^2}{M_h^2}\right)}{\sum_{b=4,5} m_b \operatorname{Re}(\lambda_{2b}^*\lambda_{b2}) \tilde{B}_0\left(\frac{m_b^2}{M_h^2}\right)}. \tag{5.9}$$

We want to simplify this expression further in the limit of $m_b^2 \gg M_h^2$, starting by asymptotically approximating the loop functions G_h and $\tilde{B}_0$:

$$G_h(x) \sim -\frac{1}{x} \ (x \to \infty) \tag{5.10}$$

$$\tilde{B}_0(x) \sim 1 - \frac{1}{x} \ (x \to \infty). \tag{5.11}$$

In this limit, the contribution to a_μ is given by:

$$\begin{aligned} \delta a_\mu^h &\sim \frac{m_\mu}{32\pi^2 M_h^2} \sum_{b=4,5} m_b \operatorname{Re}(\lambda_{2b}\lambda_{b2}) \frac{M_h^2}{m_b^2} \\ &= \frac{m_\mu}{32\pi^2} \sum_{b=4,5} \frac{1}{m_b} \operatorname{Re}(\lambda_{2b}\lambda_{b2}). \end{aligned} \tag{5.12}$$

We may plug in the couplings calculated above (3.14), and obtain:

$$\delta a_\mu^h \sim \frac{\lambda_E \bar{\lambda} \lambda_L}{32\pi^2} m_\mu \left[-\left(\frac{M_E}{M_E^2 - M_L^2} + \frac{1}{M_E} \right) \frac{v}{M_L} + \left(\frac{M_L}{M_E^2 - M_L^2} - \frac{1}{M_L} \right) \frac{v}{M_E} \right]. \tag{5.13}$$

This expression already contains some interesting consequences: terms $\propto \lambda$ cancel with each other, and the remaining couplings only appear in form of the product $\lambda_E \bar{\lambda} \lambda_L$. Effectively, δa_μ^h only depends on one coupling parameter, said product of couplings. This is in agreement with findings by [9].

Next, we take a closer look at the 1-loop mass correction. Approximating $\tilde{B}_0$, this is given by:

$$\frac{m_\mu^{\text{1-Loop}}}{m_\mu} \sim -\frac{1}{32\pi^2} \frac{1}{m_\mu} \sum_{b=4,5} \left[m_b \operatorname{Re}(\lambda_{2b}^*\lambda_{b2}) \left(1 - \frac{M_h^2}{m_b^2}\right) \right]. \tag{5.14}$$

Assuming the couplings $\lambda_{b2}, \lambda_{2b}$ to be real-valued, we can compare this to (5.12) and rewrite this expression in terms of δa_μ^h:

$$\frac{m_\mu^{\text{1-Loop}}}{m_\mu} \sim \frac{M_h^2}{m_\mu^2}\delta a_\mu^h - \frac{1}{32\pi^2}\frac{1}{m_\mu}\sum_{b=4,5} m_b\lambda_{2b}\lambda_{b2}. \tag{5.15}$$

We see that in the considered limit δa_μ^h and $m_\mu^{\text{1-Loop}}$ are closely related. Plugging in the specific values of the couplings yields:

$$\frac{m_\mu^{\text{1-Loop}}}{m_\mu} \sim \frac{M_h^2}{m_\mu^2}\delta a_\mu^h - \frac{\lambda_E\lambda_L}{32\pi^2}\frac{v}{m_\mu}\left[\lambda - \bar{\lambda}\left(\frac{M_L}{M_E} + \frac{M_E}{M_L}\right)\right]. \tag{5.16}$$

We find confirmed what we already saw in section 4.3: in contrast to δa_μ^h, the 1-loop mass correction (in the asymptotic limit) not only depends on $\bar{\lambda}$, but λ as well. The couplings still only appear in product form, $\lambda_E\lambda_L\lambda$ and $\lambda_E\lambda_L\bar{\lambda}$ - we effectively remain with two coupling parameters.

To simplify following expressions, we will perform a change of variables:

$$(M_L, M_E) =: (M, cM). \tag{5.17}$$

We already used this transformation in section 4.3. We remain with a single mass scale M, the dimensionless parameter $c = \frac{M_E}{M_L}$ describes the ratio of both VLL masses. With that, δa_μ^h and $m_\mu^{\text{1-Loop}}$ become:

$$\delta a_\mu^h \sim -\frac{3\lambda_E\bar{\lambda}\lambda_L}{32\pi^2}\frac{m_\mu v}{M^2}\frac{1}{c} \tag{5.18}$$

$$\frac{m_\mu^{\text{1-Loop}}}{m_\mu} \sim -\frac{3\lambda_E\bar{\lambda}\lambda_L}{32\pi^2}\frac{v}{m_\mu}\frac{M_h^2}{M^2}\frac{1}{c} - \frac{\lambda_E\lambda_L}{32\pi^2}\frac{v}{m_\mu}\left[\lambda - \bar{\lambda}\frac{c^2+1}{c}\right]. \tag{5.19}$$

In the limit $M^2 \gg M_h^2$, the first summand in (5.19) can be neglected, the remaining expression is:

$$\frac{m_\mu^{\text{1-Loop}}}{m_\mu} \sim -\frac{\lambda_E\lambda_L}{32\pi^2}\frac{v}{m_\mu}\left[\lambda - \bar{\lambda}\frac{c^2+1}{c}\right]. \tag{5.20}$$

Carrying out (5.9), we obtain:

$$\delta a_\mu : \frac{m_\mu^{\text{1-Loop}}}{m_\mu} \sim \frac{3\bar{\lambda}}{c\left[\lambda - \bar{\lambda}\frac{c^2+1}{c}\right]}\frac{m_\mu^2}{M^2}. \tag{5.21}$$

We find that in the limit of large masses, C_{VLL} is given by:

$$C_{\text{VLL}} \sim \frac{3\bar{\lambda}}{c\left[\lambda - \bar{\lambda}\frac{c^2+1}{c}\right]} = -\frac{3}{c\left[\frac{c^2+1}{c} - \frac{\lambda}{\bar{\lambda}}\right]}. \tag{5.22}$$

This verifies (5.8): C_{VLL} is a mass-independent constant, only depending on dimensionless couplings and ratios of masses.
The following plot shows $\delta a_\mu{:}\frac{m_\mu^{\text{1-Loop}}}{m_\mu}$ for some exemplary values of $\lambda/\bar{\lambda}$ and c. Dashed grey lines represent the theoretical prediction (5.21) obtained from the limit of large masses. We see that the approximation can hold down to masses of $2,3 \cdot M_h$.

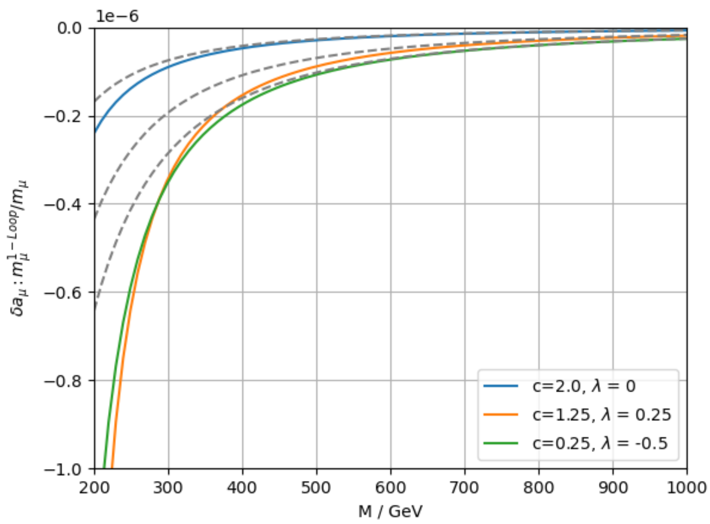


Figure 5.2: $\delta a_\mu{:}\frac{m_\mu^{\text{1-Loop}}}{m_\mu}$ for chosen values of λ, c. We fixed $\bar{\lambda} = 0.5$.

Note that this graph only displays the behaviour with respect to $m_\mu^{\text{1-Loop}}$. We saw in section 4.3 that in the range of smaller masses $M \lesssim 3$ TeV (roughly), the tree-level mass correction yields significant contributions as well and usually dominates. Examining the relation between corrections to a_μ and the muon mass when considering both $m_\mu^{LE} + m_\mu^{\text{1-Loop}}$ will be the goal of the following section.

5.3 Numerical studies for the general case

From now on we also want to take the tree-level mass correction into account, so we redefine C_{VLL} accordingly:

$$C_{\text{VLL}} := \delta a_\mu : \frac{m_\mu^{LE} + m_\mu^{\text{1-Loop}}}{m_\mu} \cdot \frac{M^2}{m_\mu^2} \tag{5.23}$$

We saw in the last section that in the asymptotic limit effectively only two independent couplings remain, $\lambda_E\lambda\lambda_L$ and $\lambda_E\bar{\lambda}\lambda_L$. We assume varying $\lambda_{E,L}$ independent of $\lambda, \bar{\lambda}$ still only results in negligible changes and fixate them, in agreement with [9], according to electroweak precision data [7]. This roughly corresponds to:

$$\frac{\lambda_E v}{M_E} \lesssim 0.03, \quad \frac{\lambda_L v}{M_L} \lesssim 0.04. \tag{5.24}$$

We always work with the maximum possible value for λ_E, λ_L which fulfills these boundaries., i.e. the one given by the largest masses $M = M_L, cM = M_E$.

We expect the tree-level correction to dominate in the region of small masses, so we can only hope that results of the preceding section apply for bigger masses.
Following diagrams display $C_{\text{VLL}}(M)$ for different values of $c, \lambda, \bar{\lambda}$. Dashed grey lines represent the analytical result of (5.22).
We see that C_{VLL} converges in every case, only figure (5.7) shows somewhat of a different behaviour, we will try to explain this in a moment.
The estimation carried out in 4.3 is useful for the most cases. It seems to fail for the cases of $c = 0.1$ and $c = 5.0$. The threshold from which $m_\mu^{\text{1-Loop}}$ dominates is much smaller in the case of $c = 5.0$, and much larger in the case of $c = 0.1$. This was expected, since we assumed c to be $c \simeq 1$ in order to carry out the estimate. We can also observe that the curves for those two cases seem to only vary little for different couplings $\lambda, \bar{\lambda}$.
Significant changes in behaviour appear in figure (5.7). Although this is not immediately obvious when looking at the plot, it is reasonable to assume that all curves converge as well according to (5.22), only over a wider range of masses. We may explain this behaviour as follows: Solving the relationship between tree-level and 1-loop-mass correction (4.21) for $m_\mu^{LE} = m_\mu^{\text{VLL}}$ yields a mass scale $M \sim \left(\frac{\lambda}{\bar{\lambda}} - \frac{c^2+1}{c}\right)^{-1/2}$. In figure (5.7), the case of $\frac{\lambda}{\bar{\lambda}} - \frac{c^2+1}{c} \simeq 0$ may occur, following our estimation we would expect a very large range of masses in which the tree-level correction dominates.

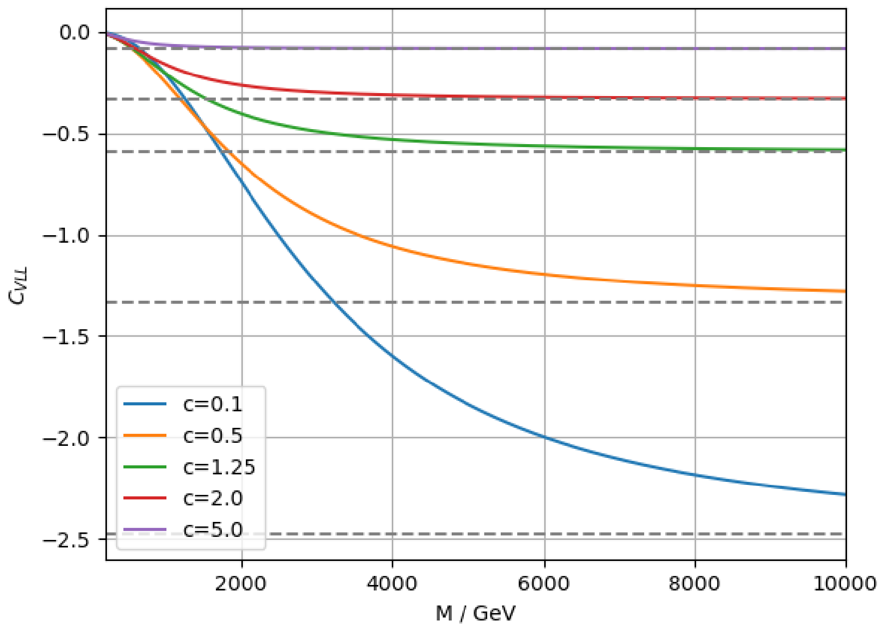


Figure 5.3: $C_{\text{VLL}}(M)$ with $\lambda = 1.0, \lambda_b = -0.5$

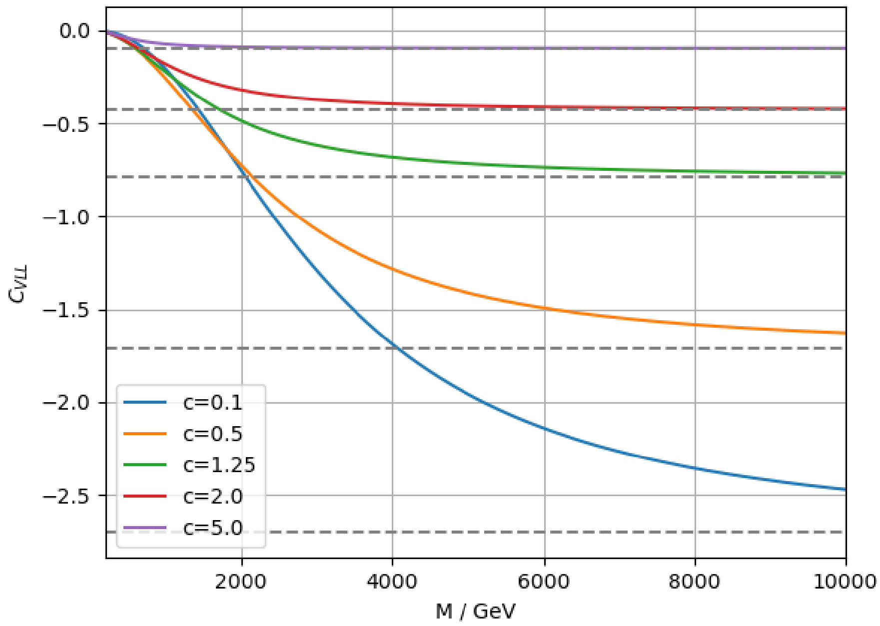


Figure 5.4: $C_{\text{VLL}}(M)$ with $\lambda = 0.5, \lambda_b = -0.5$

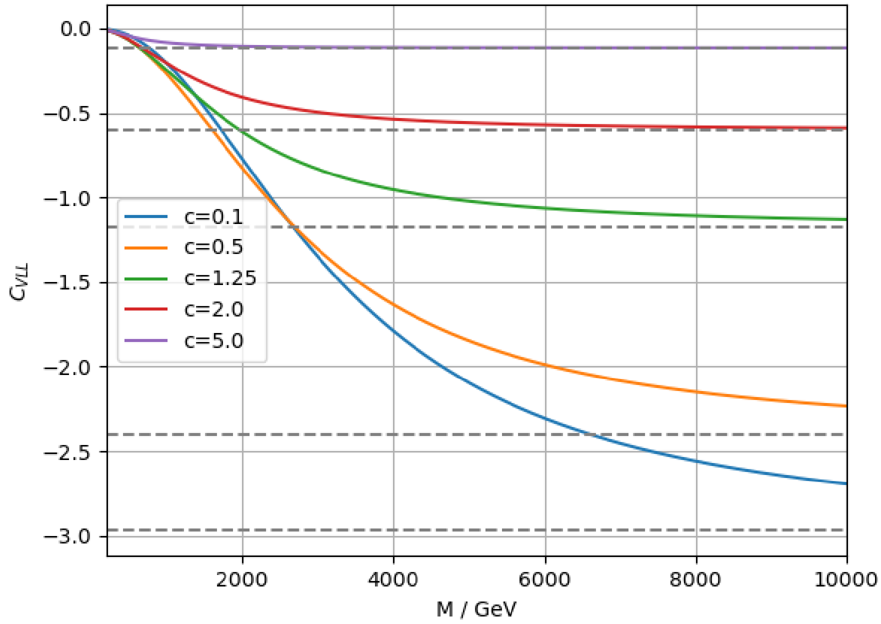


Figure 5.5: $C_{\text{VLL}}(M)$ with $\lambda = 0, \lambda_b = 0.5$

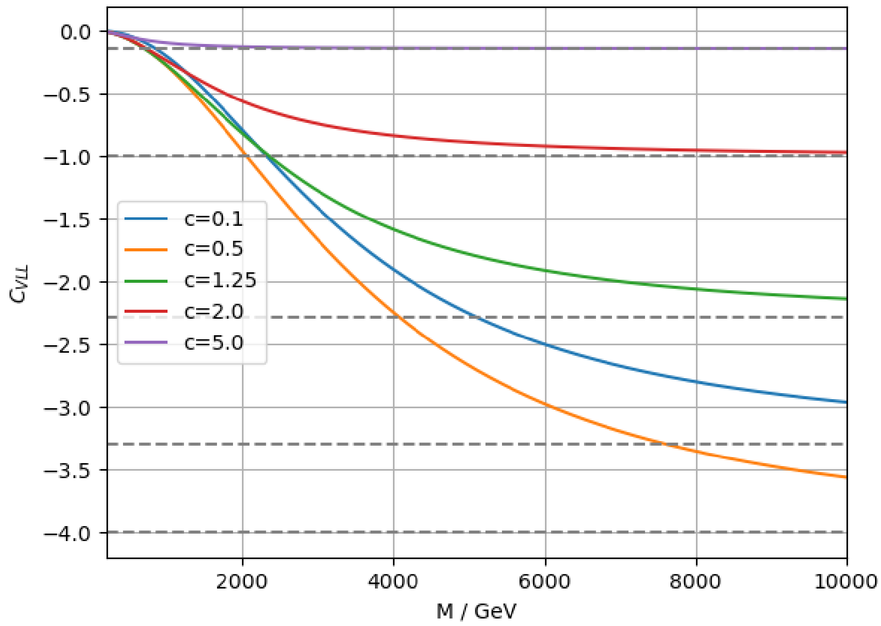


Figure 5.6: $C_{\text{VLL}}(M)$ with $\lambda = 0.5, \lambda_b = 0.5$

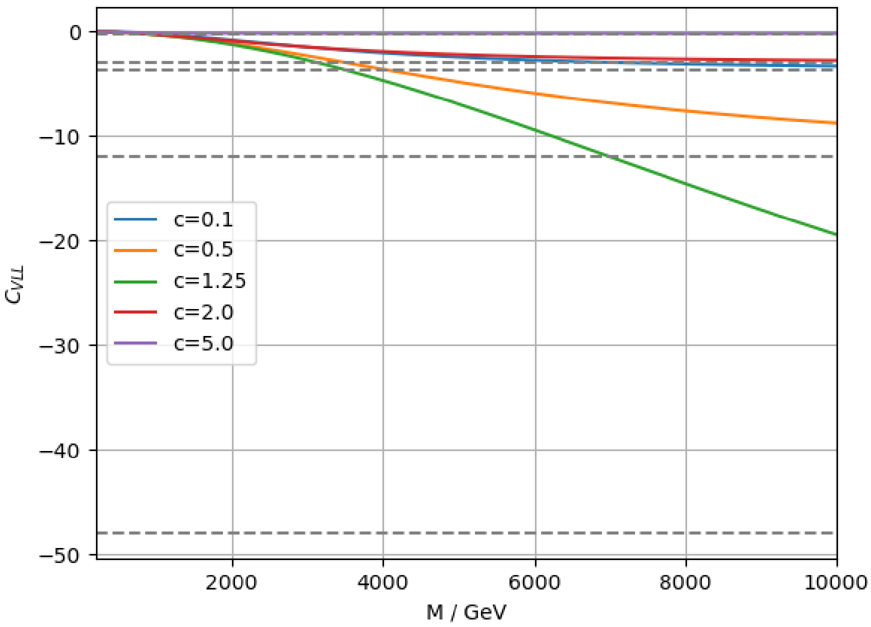


Figure 5.7: $C_{\mathrm{VLL}}(M)$ with $\lambda = 1.0, \lambda_b = 0.5$

6 Summary and Outlook

We began this thesis with a short introduction on the description of charged leptons by the Standard Model of particle physics, focusing in particular on the Higgs mechanism. This mechanism gave rise to particle masses, through spontaneous symmetry breaking of the $\mathrm{SU}(2)\times\mathrm{U}(1)$ symmetry underyling the Standard Model.
We introduced the model of vector-like leptons, and applied the tools acquired to understand contributions to the muon mass appearing through mixing with VLL.
After examining tree-level contributions, we calculated the mass correction at 1-loop level. Comparing both contributions, we found that the 1-loop correction dominates in the limit of large VLL masses.
Using this fact, we were able to show that a close connection between corrections to a_μ and mass corrections holds true in this limit, $\delta a_\mu \sim \mathcal{O}(\delta m_\mu^{\mathrm{VLL}}/m_\mu)\frac{m_\mu^2}{M_{\mathrm{VLL}}^2}$.
We showed numerically that this connection also holds true in the region of large VLL masses when the tree-level correction is taken into account. For smaller masses, in the region where the tree-level contribution dominates, deviations from this relation occur. We estimated the threshold that separates both regions to be $\simeq 3$ TeV for most cases.

In above investigations, we restricted ourselves to studying contributions to a_μ originating from the Higgs diagram. Because of this, we were not able to make any specific prediction on allowed values for the free parameters of the model, the contributions originating from $W-$ and $Z-$interactions are by no means negligible. Taking those into consideration as well would allow a direct comparison with the experimental value for a_μ.
We did not discuss vector-like neutrinos and quarks, although both of them are also well established candidates for BSM physics. One may also consider combining the VLL model with other BSM models, for example the two-Higgs-doublet model (see e.g. [8]).
Only time, and future experiments, will tell which models suit best our description of nature.

7 Bibliography

[1] Particle Data Group (PDG). 2021. URL: https://pdg.lbl.gov/2021/listings/rpp2021-list-higgs-boson.pdf (visited on 06/14/2021).

[2] Particle Data Group (PDG). 2021. URL: https://pdg.lbl.gov/2021/listings/rpp2021-list-electron.pdf (visited on 06/14/2021).

[3] Particle Data Group (PDG). 2021. URL: https://pdg.lbl.gov/2021/listings/rpp2021-list-muon.pdf (visited on 06/14/2021).

[4] Particle Data Group (PDG). 2021. URL: https://pdg.lbl.gov/2021/listings/rpp2021-list-tau.pdf (visited on 06/14/2021).

[5] B. Abi et al. "Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm". In: *Phys. Rev. Lett.* 126 (14 Apr. 2021), p. 141801. DOI: 10.1103/PhysRevLett.126.141801. URL: https://link.aps.org/doi/10.1103/PhysRevLett.126.141801.

[6] Peter Athron et al. *New physics explanations of a_μ in light of the FNAL muon $g-2$ measurement*. 2021. arXiv: 2104.03691 [hep-ph].

[7] J. Beringer et al. "Review of Particle Physics". In: *Phys. Rev. D* 86 (1 July 2012), p. 010001. DOI: 10.1103/PhysRevD.86.010001. URL: https://link.aps.org/doi/10.1103/PhysRevD.86.010001.

[8] Radovan Dermisek et al. "Highly Enhanced Contributions of Heavy Higgs Bosons and New Leptons to Muon $g-2$ and Prospects at Future Colliders". In: *Phys. Rev. Lett.* 126.19 (2021), p. 191801. DOI: 10.1103/PhysRevLett.126.191801. arXiv: 2011.11812 [hep-ph].

[9] Radovan Dermíšek and Aditi Raval. "Explanation of the muong2anomaly with vectorlike leptons and its implications for Higgs decays". In: *Physical Review D* 88.1 (July 2013). ISSN: 1550-2368. DOI: 10.1103/physrevd.88.013017. URL: http://dx.doi.org/10.1103/PhysRevD.88.013017.

[10] T. Kugo. *Eichtheorie*. Springer-Verlag Berlin Heidelberg, 1997.

[11] C. Quigg. *Gauge Theories of the Strong, Weak, and Electromagnetic Interactions*. Princeton University Press, 2013.

[12] M. Schwartz. *Quantum Field Theory and the Standard Model*. Cambridge University Press, 2014.

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Erklärung

Hiermit erkläre ich, dass ich diese Arbeit im Rahmen der Betreuung am Institut für Kern- und Teilchenphysik ohne unzulässige Hilfe Dritter verfasst und alle Quellen als solche gekennzeichnet habe.

Nico Albert
Dresden, Juni 2021