TECHNISCHE UNIVERSITÄT DRESDEN

Majorana Neutrinos with Displaced Vertices and Implementation into Sherpa

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Zusammenfassung

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Deutsch:
Das Seesaw-Modell Typ I wird mit Hilfe von UFO in Sherpa eingebunden. Die Richtigkeit der Installation wird durch den Vergleich von simulierten partiellen Zerfallsbreiten mit explizit theoretisch vorhergesagten bestätigt. Ein Vertex-Versatz für hochenergetische Elementarteilchen-Zerfälle wird implementiert. Dazu wird zuerst eine pseudo-zufällige Lebensdauer im Ruhesystem des Teilchens generiert. Diese wird durch Lorentz-Transformation in das Laborsystem übertragen. Dort wird mit dem Impuls des Teilchens der Vertex-Versatz bestimmt.

Abstract
English:
The inclusion of the Seesaw model type I into Sherpa using UFO is explained. Installation is validated by comparing simulated partial and total decay widths for Majorana neutrinos to analytic theoretical predictions. Vertex displacements are implemented for high energy elementary particle decays the following way. A pseudo-random rest frame lifetime is generated according to the physical distribution. It is boosted into the laboratory system and using the particle's momentum the decay position is estimated.

Contents

1 Introduction

Particle physics is a part of science trying to explain the most basic particles and their interactions. The Standard Model of elementary particles describes this particle world nicely universally. The Standard Model can be described by its Lagrangian density function, which leads to all predicted physics laws. The Standard Model leaves some parameters unspecified. Those parameters comprise coupling strengths and particle masses, they have to be taken from experiments.
However, the standard model has limitations: neutrinos are assumed as massless, but there have been found neutrino ascillations. There are three neutrino flavours known. Neutrinos can change from one to another flavour depending on the length they propagated. This only works if all three neutrinos have pairwisely different masses. So at least two of them must have a nonzero mass. Thus, there is new exciting physics beyond the Standard Model. Why are the neutrino masses at least six orders of magnitude smaller than the next heavier known particle, the electron?
For an explanation how neutrino masses can be so small there are different theories, one of which is the Seesaw mechanism [1]. It is an extension of the Standard Model. One Seesaw model, namely type one, predicts additional heavier right-handed neutrinos. Those are Majorana particles, i.e. they are their own antiparticles. For a validation of this theory it is necessary to search for these Majorana neutrinos. The masses of these theoretical particles are not known just like it was the case about the Higgs particle. One even doesn't know if they exist at all. Some regions of combinations of masses and couplings are already excluded by former experiments. A wide mass region at higher masses is not accessible because our experiments are limited in their energy. New larger accelerators will extend these limits again and again. Already with the existing experiments, e.g. at the Large Hadron Collider near Geneva could probably extend the search regions.
Particle reactions at high energy physics (HEP) experiments mostly take place almost exactly at the crossing point of the beams. Especially the hard processes are close to being at a single point. Majorana neutrino's decay widths are expected to be of the order m^5 with m being the particle's rest mass [2]. At masses around 1 GeV and assuming light speed waylengths in the order of millimeters are predicted. The main way to see a sign for existing Majorana particles would be lepton-number violation.
The comparison between theory and measured data can be done with help of simulations.

Production and decay of Majorana neutrinos can be simulated using the program Sherpa [3]. Sherpa is a C++ software package developed at TU Dresden for simulation of high energy particle interactions. The simulation results can help to find ways to examine the theory and to compare experimental data to certain theories.
This study regards the simulation of Majorana neutrinos using Sherpa. First, an introduction into particle decays, event generation and the Seesaw model is given. Then the installation of Sherpa with the model file and the implementation of vertex displacements are discussed. Finally, simulation results are compared to theory and displacement results are shown.

2 Theoretical Background

2.1 Decays

Decays can happen to bound states like nuclei and hadrons or decays can be the spontaneous transition of unstable elementary particles into other elementary particles. For most decaying particles, e.g. nuclei, the decay probability in a small time interval is proportional to the length of the time interval. In a formula this is

$$dN = -\lambda \cdot N(t) \cdot dt \tag{2.1}$$

with the decay rate λ and dN being the change of particle number within the time interval dt. This differential equation has solutions

$$N(t) = N_0 e^{-\lambda t}. \tag{2.2}$$

The free parameter N_0 is set to the number of particles at the time $t = 0$. The solution $N(t)$ describes the expectation value of remaining particles after the time t. Regarding a single particle, $N_0 = 1$ and $N(t)$ is the probability of not being decayed until time t. The half-life time of the particle then is $t_{\frac{1}{2}} = \frac{ln2}{\lambda}$. The average lifetime is for such exponential decays defined as the time until a fraction $\frac{1}{e}$ of the original particles is expected to remain:

$$\tau = \int_0^\infty t \cdot p(t)dt = \frac{1}{\lambda} = t(\frac{1}{e}N_0). \tag{2.3}$$

According to Heisenberg's uncertainty principle, the energy uncertainty of an unstable quantum state is inversely proportional to the time it exists. As energy and mass are physically equivalent, an unstable particle's mass is not sharp but uncertain. This width of the mass peak is called decay width because it is connected to the particle's decaying behavior $\Gamma = \frac{1}{\tau}$ [1]. If we treat decay processes as $1 \rightarrow n$ scattering processes with initial state i, we can calculate the partial decay width Γ_{fi} for every possible process $|i\rangle \rightarrow |f\rangle$. The total decay width is then $\Gamma_i = \sum_f \Gamma_{fi}$ and the average lifetime is $\tau = \frac{1}{\Gamma_i}$. The branching fraction for a single process $i \rightarrow f$ is $B_f = \frac{\Gamma_{fi}}{\Gamma_i}$. If we consider the initial and final state of an elementary particle process

[1] In this thesis all values that are not given in SI units explicitly are meant in natural units basing on 1GeV and $c := \hbar := 1$.

as vectors, then the transition between them can be expressed through the Lorentz-invariant matrix element M_{fi}. Then is $M_{fi} = \langle f|H'|i\rangle$ with H' the transition part of the Hamiltonian. This matrix element can be calculated for a process regarding the Feynman rules for the process. With Fermi's golden rule one can determine the width for a decay using the matrix element

$$\Gamma_{fi} = \frac{(2\pi)^4 S}{2E_i} \int \frac{d^3\vec{p_1}}{(2\pi)^3 \cdot 2E_1} \cdots \int \frac{d^3\vec{p_n}}{(2\pi)^3 \cdot 2E_n} |M_{fi}|^2 \delta^4(p_i - \sum_{j=1}^{n} p_j),$$

where p_i are the resulting particles' 4-momenta, E_i their energies and $\vec{p_i}$ their 3-momenta. The symmetry factor S is a product of factors $\frac{1}{m!}$ for each group of m identical products. While the integral Γ is a common decay probability density for an infinite time interval, leaaving out the integrals leads to decay probabilities regarding the decay products' momenta. Fermi's golden rule concerns momenta and time in the center-of-mass frame (cms) of the process. For decays it is equivalent to the incoming particle's inertial frame of reference. If we got the decay time of a particle in its cms, we can boost its lifetime into the lab system. Then we can determine its 4-dimensional point of decay in the lab system using the particle's momentum.

2.2 Event Generation with Sherpa

In high-energy particle physics, experiments are done in large complex accelerators and detectors. The underlying physical laws include quantum physics and some of the rules are non-deterministic. Because of that, it is not possible to get the inner high-energy reactions, called hard scattering processes, directly from the measurement results. Instead one can simulate possible realized events according to a certain theory with computers and then compare the results to the experimental ones. Particle physics theories, as the Standard Model of elementary particles, describe how to calculate probabilities for possible results of elementary processes. We want to get the probability distribution of the results of a whole HEP experiment, in which case it is technically impossible to calculate final probabilities analytically.
However, we can use Monte Carlo methods. Monte Carlo methods mean to describe the wanted variable as an expectation value of a random experiment. Then many single simulations of this experiment are done using pseudo-random numbers. According to the law of large numbers, after a large amount of realizations, the simulated probability distribution comes close to the real one. How close the simulation comes to the true value, depends on the number of realizations and can be determined regarding laws of probability. This statistical uncertainty should always be considered.
Integrals can also be determined using Monte Carlo methods. For the calculation of an integral $\int_0^1 f(x)dx$ with $f(x) \in [0,1] \, \forall x$, each value $f(x)$ is interpreted as the frequency of its argument x. The integral is equivalent to the expectation value of the so constructed random experiment. The integrator program determines many uniformly distributed pseudo-random points

$(x, r) \in [0,1]^2$. The rate of points with $r < f(x)$ gives the integral. Most one-dimensional integrals can be written in this form. For higher-dimensional problems, Monte Carlo integration is also possible. Then in the abovementioned procedure, the variable x has to be replaced by the vector $\vec{x}$. Simulating HEP events includes very high-dimensional problems. That's why Sherpa uses Monte Carlo methods.

Sherpa [3] stands for "**S**imulation of **h**igh-**e**nergy **r**eactions of **pa**rticles". It is a Monte Carlo event generator for HEP events. It is programmed in object-oriented C++. Sherpa uses Monte Carlo methods for elementary calculations such as integrals, and the whole concept of the event generator is based on the Monte Carlo idea. Sherpa takes as input parameters a certain physics model, e.g. the Standard Model, and the initial collision setup. It then calculates an arbitrary number of realizations of this setup. Every time some elementary process is not deterministic but the probability distribution is computable, one concrete result is realized using the right distribution. The contribution of all generated events is assumed to be according to the input physics and it can be compared to experiment results. If the simulation does not fit the experimental results, the model can be modified and the simulation can be tried again.

Sherpa event generation is devided into certain phases that are passed one after another while the creation of one event with each phase responsible for a given energy scale. They are divided into perturbative and non-perturbative phases. The phases are called recursively until nothing is to be calculated anymore. As depicted in Figure 2.1, the central part of event generation

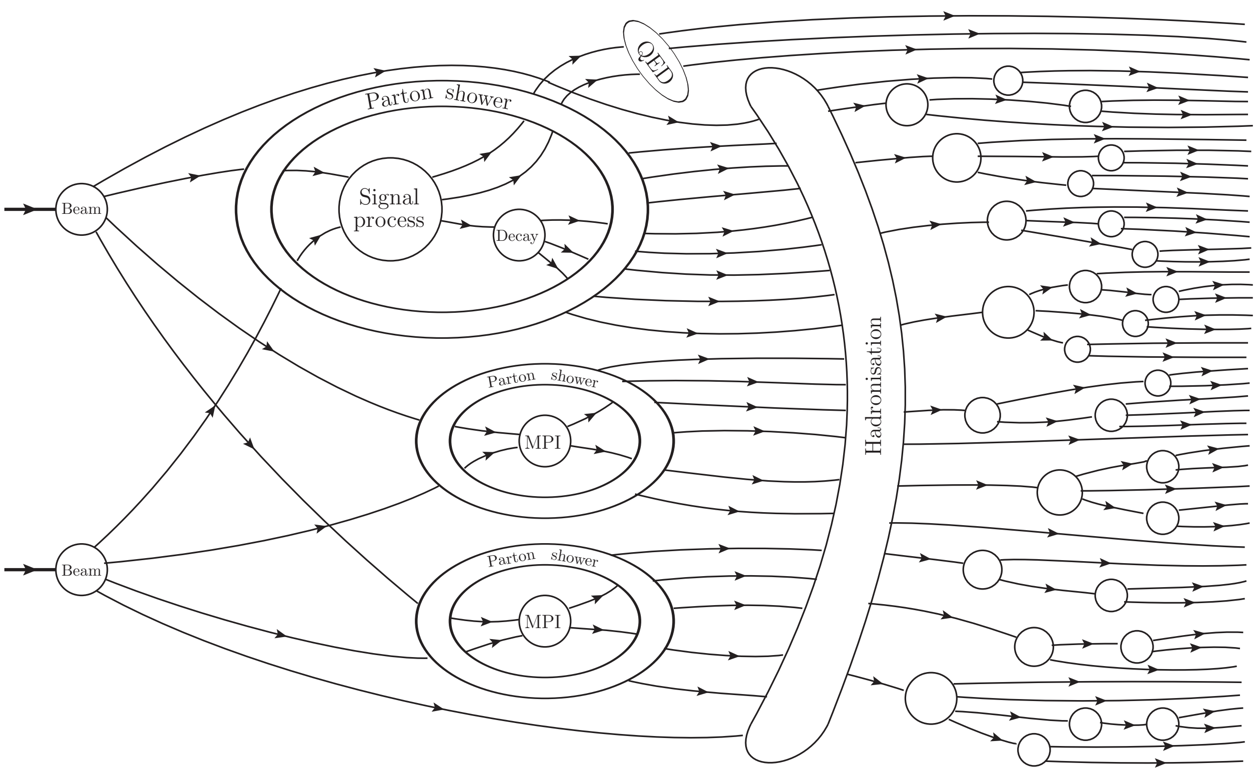

Figure 2.1: Sherpa event generation structure. [4]

is the signal process. The spectator partons out of the beam which don't participate in the signal process are not relevant for this thesis. The outgoing particles after the signal process are treated concerning elementary particle decays. Then the incoming and outgoing particles of the whole hard processes are parton showered. This means to regard QCD Bremsstrahlung. The shower is technically implemented as a single vertex with typically many incoming and outgoing particles. It is possible that particles do not interact while showering, but those get through the shower vertex as well. Some cases include the simulation of additional QED Bremsstrahlung. The whole output of the parton showers is then hadronized. There all partons form several hadrons, which is done on a single vertex again. In the end, unstable hadrons are decayed.

2.3 Seesaw Model

Three neutrino flavours are known today of which each belongs to one lepton generation. Neutrino oscillations mean that neutrinos are detected as another flavour than they had while their production [5]. Since neutrino oscillations have been observed, it is known that all three neutrino flavour must have different masses. Thus, at least two neutrino flavours' masses must be larger than zero against the Standard model's assumption that all neutrinos were massless. The large difference between those required neutrino masses and all other known much larger fermion masses seems itself not natural. The Seesaw mechanism is a way to explain neutrino masses and their smallness.

In the context of quantum field theory, a model can be described compactly by its Lagrangian. A term of the form $-M_{\mathrm{D}}\bar{\psi}\psi$ in the Lagrangian is called a Dirac mass terms as it generates the mass for a Dirac pair of particle and antiparticle. In the standard model right-handed neutrinos and left-handed anti-neutrinos have all charges equal to zero. Right-handed Dirac neutrinos are absent in the standard model because they do not even participate in weak interaction and thus they are completely sterile. A term of the form $\frac{1}{2}M_{\mathrm{M}}N_{\mathrm{R}}N_{\mathrm{R}}$ for a fermion would be a Majorana mass term because it generates the mass for a Majorama particle, i.e. a particle that is its own antiparticle. But in the standard model no such term exists. For changing neutrino's masses one can try to add a Majorana mass term to the SM Lagrangian. If the Lagrangian contains a Dirac mass term and a Majorana mass term as well which is considered in the following, then the so-called Seesaw mechanism will arise. There won't be a term $\bar{\nu_L}M_{\mathrm{M}}\nu_{\mathrm{L}}$ in the Lagrangian because this Majorana mass term for left-handed neutrinos would not be compatible with the Higgs mechanism. The relevant terms of the Lagrangian

$$-\mathcal{L}_\nu = \bar{\nu_{\mathrm{L}}}M_{\mathrm{D}}N_{\mathrm{R}} + \nu_{\mathrm{L}}M_{\mathrm{D}}\bar{N_{\mathrm{R}}} + N_{\mathrm{R}}M_{\mathrm{M}}N_{\mathrm{R}} \tag{2.4}$$

can be written as a product of matrices

$$-\mathcal{L}_\nu = \begin{pmatrix} \bar{\nu_{\mathrm{L}}} & \bar{N_{\mathrm{R}}} \end{pmatrix} \begin{pmatrix} 0 & M_{\mathrm{D}} \\ M_{\mathrm{D}} & M_{\mathrm{M}} \end{pmatrix} \begin{pmatrix} \nu_{\mathrm{L}} \\ N_{\mathrm{R}} \end{pmatrix}, \tag{2.5}$$

where the two parameters M_{D} and M_{M} remain to be taken from experiment. Then neutrino masses are eigenvalues of this mass matrix. The exact eigenvalues are

$$m_{1,2} = \frac{M_M}{2} \pm \sqrt{\frac{M_M^2}{4} - M_D^2}. \tag{2.6}$$

For the explanation of small neutrino masses the smaller solution m_2 must be small. This is possible if the value of the root $\sqrt{\frac{M_M^2}{4} - M_D^2}$ is close below $\frac{M_M}{2}$, which means that $M_M \gg M_D$. With M_M being much bigger than M_D, equation 2.6 laeds to

$$m_2 \equiv m_\nu \approx \frac{M_{\mathrm{D}}^2}{M_{\mathrm{M}}} \ll M_{\mathrm{D}} \tag{2.7}$$

$$m_1 \equiv m_N \approx M_{\mathrm{M}} \gg M_{\mathrm{D}}. \tag{2.8}$$

This would naturally explain the smallness of neutrino masses compared to all the other Dirac masses. But at the same time it would lead to additional heavy, right-handed neutrinos N_R. The smaller the left-handed neutrino masses are, the heavier are the right-handed ones, which explains the term “Seesaw mechanism”. While there are at least two neutrinos needing a small mass, the Seesaw mechanism will give two or more additional right-handed heavy neutrino flavours [2]. There are several models using seesaw mechanisms. The one regarded in this thesis is the most natural one and is called Seesaw model Type I [1]. It assumes Seesaw mechanisms for all three known neutrino flavours and thus adds three heavy neutrinos to the SM particle content.

Heavy neutrinos can mix with each other and light neutrinos, so the gauge states are a linear combination of the mass states. For Seesaw model type I this means

$$\nu_{aL} = \sum_{m=1}^{3} U_{am}\nu_{mL} + \sum_{n=1}^{3} V_{an}N_{nL}^C, \tag{2.9}$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L + \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix} \cdot \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_L^C. \tag{2.10}$$

The matrix U_{am} alone describes the mixing between light neutrinos as it is present in the SM. The matrix V_{an} describes the mixing between heavy and light neutrinos. In this thesis all diagonal mixing will be considered. Then the gauge states are equivalent to the mass states. As M_M is not fixed by experiment, heavy neutrino masses are not defined yet. The Majorana masses are taken as free parameters. Concrete cross sections can then be expressed in terms of these masses. For that one computes Feynman diagrams using the Feynman rules out of the model's Lagrangian. Just like Dirac neutrinos, Majorana neutrinos participate only in weak interaction. They couple via neutral current, charged current and with the Higgs particle. One can replace any neutrino or antineutrino by a majorana neutrino in the neutrinos' couplings. Majorana neutrinos can act like anti-neutrinos since they are their own antiparticle. Every process where an anti-neutrino was replaced by a Majorana neutrino violates lepton number conservation.

For sufficiently large masses, heavy neutrinos decay into $W^{\pm}$, Z or H and corresponding leptons. For lower masses Majorana neutrinos decay via three-body decays, where at least one product is a lepton. With assumed diagonal mixing, the lepton always belongs to the same lepton generation as the original particle. The other products are either a pair of leptons or a pair of quarks.

3 Implementation

3.1 Running Sherpa

The Monte Carlo event generator Sherpa has been installed from source using the version control subversion. After a checkout, all source code as well as some examples and make files containing the information for configuring, making and installing are available in the install directory.

A shell script for compilation was written. It automatically does everything needed to run an updated local Sherpa version after changes in code: reconfiguring, compiling and installing Sherpa and setting up the Seesaw model.

The event generation parameters for a single Sherpa run are written to a run file. Those parameters include e.g. the number of events to generate and the choice of the calculation methods. They are written in human-readable code. Additionally, in the run card the hard processes are chosen. Beginning with these processes, events are created. Sherpa is started from the working directory where the run card is saved, one can also start Sherpa from an arbitrary location and give the run card location as a parameter. With help of the run card, one can easily run simulations with different input parameters, save settings and communicate them. An example run card can be found in Appendix A.

3.2 UFO

UFO is a program for the calculation of Feynman rules [6]. It only needs the Lagrangian of a theory and is able to derive all Feynman rules. These can then be used for other applications. Sherpa provides an interface to UFO [7]. There is a python script which includes a given UFO model into Sherpa automatically. For simulations based on the model it only needs to be included in the the run section of the run-card. In an additional `ufo` section in the run-card the model parameters are specified. Specifically for the BSM model Seesaw Type I, the UFO file "Seesaw_TypeI_UFO" is available [8]. This is used for the simulations and for tests of implementation. When using the Seesaw model type I UFO file, the model parameters include the heavy neutrino masses, several coupling parameters and the heavy neutrino mixing matrix. The mixing matrix was set all diagonal for this thesis.

3.3 Vertex Displacement

In the current Sherpa version, decays of elementary particles have no vertex displacement. Unstable particles with high energies are assumed to decay instantly. This assumption is valid in general because high energy leads to short lifetimes. For neutral heavy leptons the expected lifetimes at a specific mass range below the W mass motivate the calculation of exact decay vertex positions.
The relevant part of the program is the "hard decay handler" which performs elementary particle decays. The task was to include vertex offsets in the hard decay handler.
The hard decay handler creates decay trees as far as they are elementary decays. For that purpose in the hard decay phase of the simulation the function `FillDecayTree()` is called recursively for every unstable particle. First a decay channel is chosen according to the beforehand calculated decay table and using pseudo-random numbers. At this point a decay vertex with new outgoing particles is created. The 4-position of this decay vertex is initially set to $(0, 0, 0, 0)$. This position shall be changed regarding the physics beyond the process. Therefore, a new function `SetPosition()` has been implemented.
The function `SetPosition()` (see Appendix B) includes the time-spatial displacement for one single decay vertex. Knowing the flavour of the decaying particle, first a lifetime is generated as follows. According to the decay law, the probability density of lifetimes is

$$\frac{dp}{dt}(t) = \frac{1}{\tau} e^{-\frac{t}{\tau}},$$

where τ is the average lifetime. Using a pseudo-random number r that is confirmly distributed in the interval $(0, 1)$, using the inversion method lifetimes are calculated as

$$t_{\text{unboosted}}(r) = -\tau \cdot ln(1 - r).$$

This time is the unboosted lifetime is valid in the particle's rest frame. The relativistic factor is calculated as $\gamma = \frac{E}{m}$. Then the particle's lifetime in the lab system is given by $t_{\text{lab}} = \gamma \cdot t_{\text{unboosted}}$. With the known momentum vector of the particle, the propagated distance is calculated. The lifetime and distance are added to the production position and give the 4-vector decay position. The ongoing simulation works with this displaced vertex, i.e. following vertex positions are set in relation to this.

4 Results

4.1 Heavy Neutrino Production at the LHC

Sherpa is tested by regarding several features of the model. For these tests typical setups for the LHC are used. Two proton beams collide at center of mass energies of typically 13 TeV. As example production process $p_1p_2 \rightarrow N_1\bar{\nu}_e$ waas chosen, where p_i stands for any parton. Heavy neutrino masses are set to several values above m_W and m_Z to gian $1 \rightarrow 2$ decays or even below for $1 \rightarrow 3$ decays. Figure 4.1 shows the angle distribution of the N_1 decay products W^+

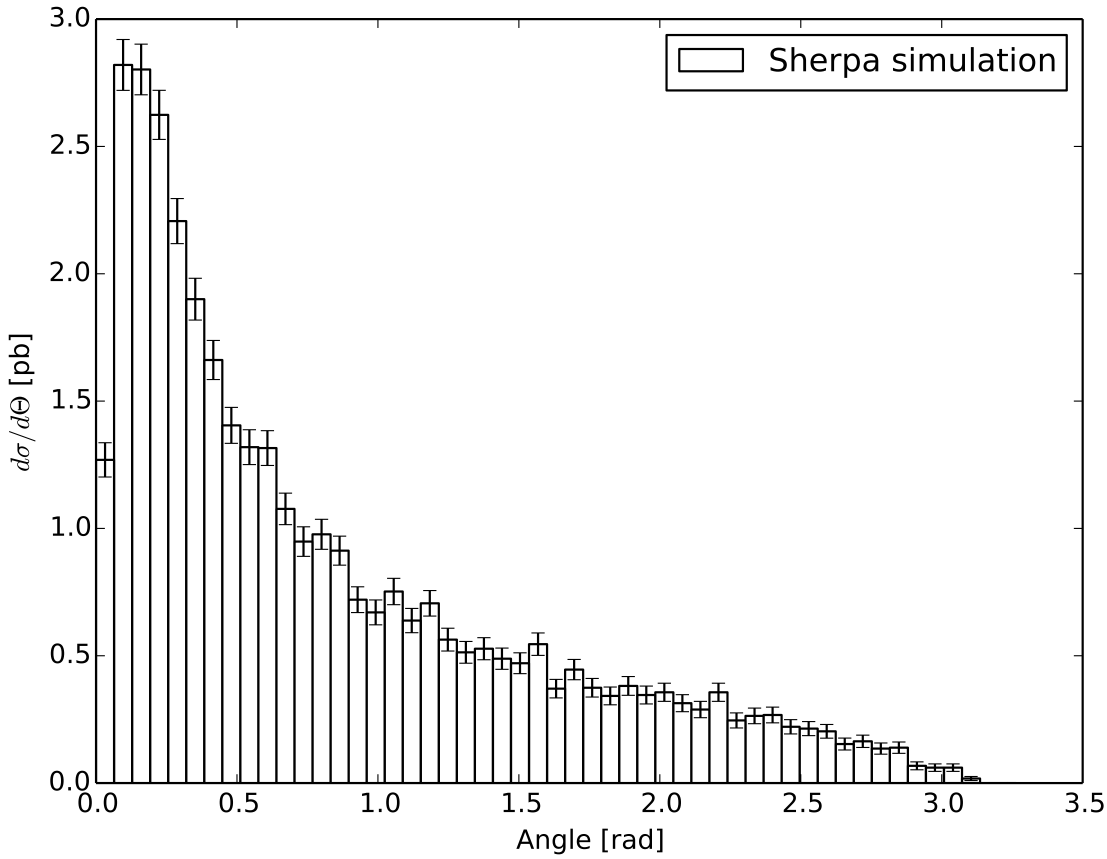

Figure 4.1: Angle between W^+ and e^- momenta in the laboratory system for 10000 simulated events at 8TeV with $m(N_1) = 100.0$GeV

and e^-. One observes a maximum close above the angle zero. For larger angles the frequency falls until it arrives at zero for the angle π. It is clear that angles have to be in the range from 0 to π. In the decaying particle's inertial system the two products always should move in opposite directions. Because of the Majorana neutrino typically moving fast compared to the relative velocity between the decay products, the resulting angle in the laboratory system is small in most cases.

Figure 4.2 depicts the frequencies of invariant masses of the heavy neutrino N_1. The curve

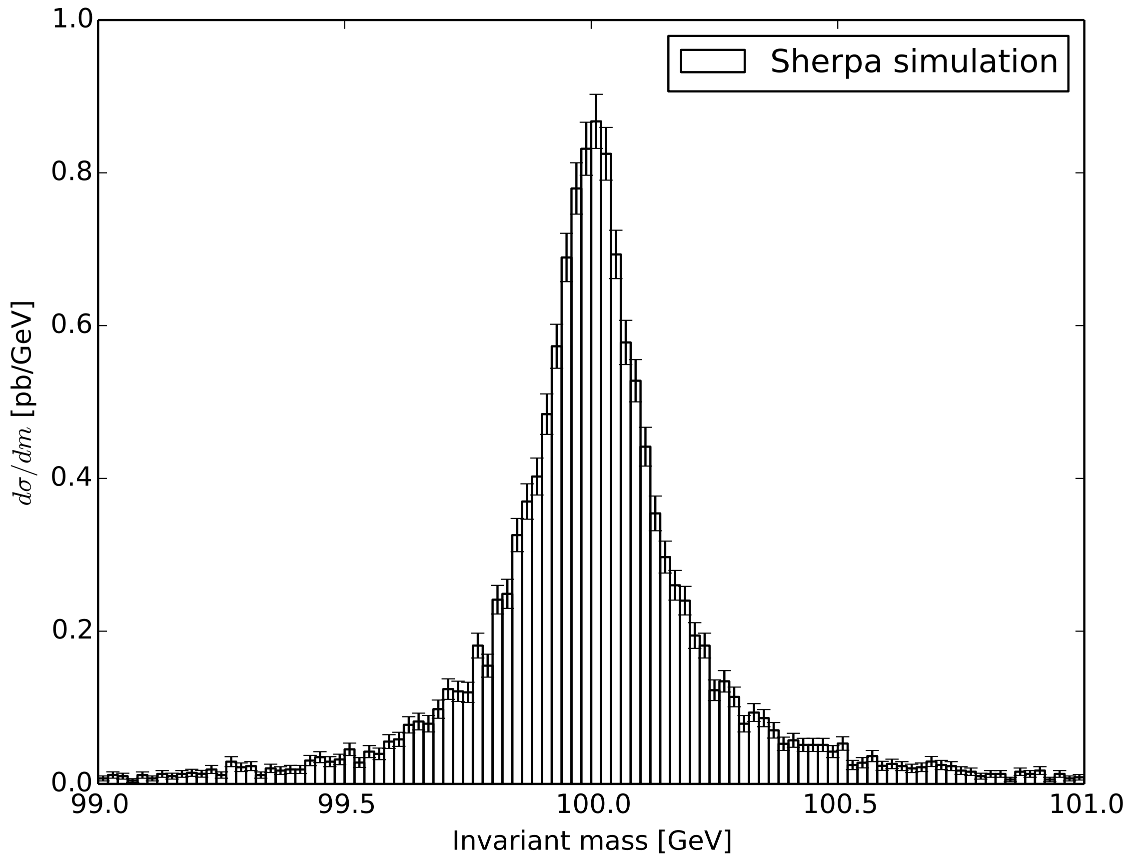

Figure 4.2: Invariant mass distribution of N_1 for decay products Z and ν_e; 10000 events at 8TeV with $m(N_1) = 100\text{GeV}$

forms a peak at the chosen invariant mass of the decaying particle. One can see well that the mass peak is not sharp but has a nonzero width Γ. Obvously the width comes close to the expected decay width $\Gamma = 0.22\text{GeV}$.

Calculating the widths for $1 \rightarrow 3$ decays with $m_N < m_W$ is technically more difficult than the ones at higher masses because Sherpa has to integrate them. Thus the validation of Sherpa's output at those lower masses is a favorable way to check if errors occur. For $m(N_1) < m_W$ decays are $1 \rightarrow 3$ decays. That's why an other calculation method is required. For a validation of that Figure 4.3 also shows simulated invariant masses of a heavy neutrino but with $m(N_1) = 70\text{GeV}$ for the decay $N \rightarrow e^- u\bar{d}$. The peak has approximately the width $\Gamma = 0.0019\text{GeV}$, which

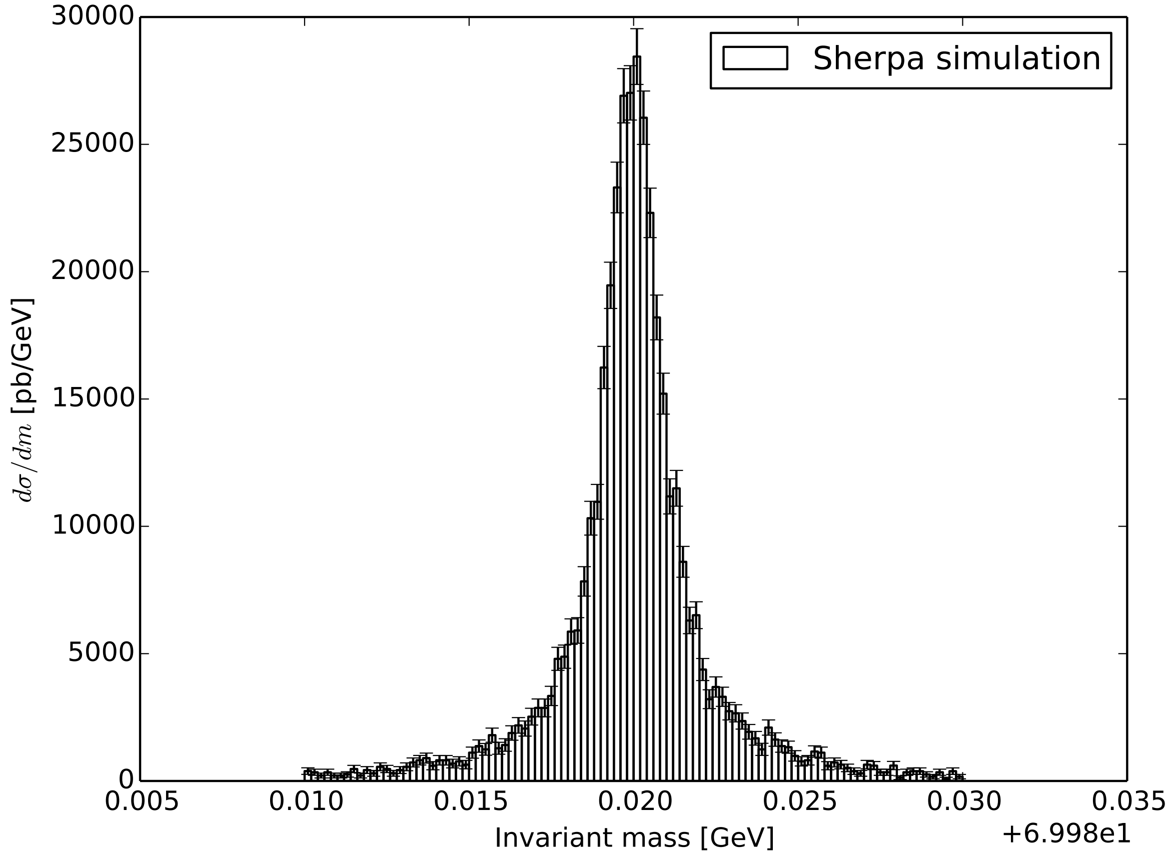

Figure 4.3: Invariant mass distribution of N_1 for decay products e^-, u and $\bar{d}$; 10000 events at 13TeV with $m(N_1) = 70\text{GeV}$

matches the theoretical prediction. The calculation of widths for three body decays seems to work appropriately.

4.2 Validation of Decay Widths

For a validation that Sherpa is working the right way, some theoretical results shall be compared to Sherpa output. Predictions for decay widths of heavy neutral leptons for several masse can be found in [2]. Those predictions will be used for Sherpa validation. At masses $m_N > m_W$ these are given as

$$\Gamma^{lW_L} = \frac{{g_w}^2}{64\pi {M_w}^2} |V_{lN_1}|^2 {M_{N_1}}^3 (1-\mu_W)^2 \tag{4.1}$$

$$\Gamma^{lW_T} = \frac{{g_w}^2}{32\pi} |V_{lN_1}|^2 M_{N_1} (1-\mu_W)^2 \tag{4.2}$$

$$\Gamma^{\nu_e Z_L} = \frac{{g_w}^2}{64\pi {M_w}^2}|V_{lN_1}|^2 {M_{N_1}}^3(1-\mu_Z)^2 \tag{4.3}$$

$$\Gamma^{\nu_e Z_T} = \frac{{g_w}^2}{64\pi}|V_{lN_1}|^2 M_{N_1}(1-\mu_Z)^2, \tag{4.4}$$

where $\mu_W = \frac{{M_W}^2}{{M_{N_1}}^2}$.
Summation over all possible final states implies the theoretical total width

$$\Gamma_{N_1} = \sum_l (2\Gamma^{lW_L} + 2\Gamma^{lW_T} + \Gamma^{\nu_e Z_L} + \Gamma^{\nu_e Z_T}). \tag{4.5}$$

At heavy neutrino masses $m_N \ll m_W$ the partial decay widths for leptonic decays can be approximated by

$$\Gamma^{l_1 l_2 \nu_{l_2}} = \frac{G_F^2}{192\pi^3}|V_{l_1 N_1}|^2 m_{N_1}^5 \tag{4.6}$$

$$\Gamma^{\nu_1 l_2 l_2} = \frac{G_F^2}{96\pi^3}|V_{l_1 N_1}|^2 m_{N_1}^5(\alpha_1 + \delta_{l_1 l_2}\alpha_2) \tag{4.7}$$

$$\Gamma^{\nu_{l_1}\nu\nu} = \frac{G_F^2}{96\pi^3}|V_{l_1 N_1}|^2 m_{N_1}^5. \tag{4.8}$$

Other decay channels of heavy neutrinos with $m_N \ll m_W$ are given as ratios of conversions into mesons. Those are not regarded within this comparison because a seperated hadronization would be needed in the simulation, which is not given. The chosen channels are taken as representative for a validation.
For the calculation of the theoretical decay widths the same parameters as used by the UFO model were used. Those include $\alpha_{\text{QED}} = \frac{1}{127.944}$, $G_{\text{F}} = 1.17471 \cdot 10^{-5}$, $m_W = 79.9544\text{GeV}$ and $m_Z = 91.1876\text{GeV}$. Therefrom $g_W = m_W \cdot \sqrt{4 \cdot \sqrt{2}G_{\text{F}}} \approx 0.65$ and $\sin^2\Theta_W = 1 - \frac{m_W^2}{m_Z^2} \approx 0.23$ are determined. The theory for $m_N \ll m_W$ is given to be valid in a range of about $0.1\text{GeV} < m_N < 5.0\text{GeV}$. The given lower example masses were chosen to compare a mass range that is as wide as possible. As the factors α_1 and α_2 in Formula 4.7 are not given explicitly, they were calculated out of the Sherpa values giving $\alpha_1 = 0.12555097 \pm 8 \cdot 10^{-8}$ and $\alpha_2 = 0.46326883 \pm 2.7 \cdot 10^{-7}$. Thus the partial widths for $N_1 \to \nu_1 l_1 l_2$ in table 4.1 have to be understood as relative values. Special care has to be taken concerning the summation of terms with the form as given in equations 4.6 and 4.7. Summands are zero if the production of the respective products is impossible due to a too low mass. Equation 4.8 already stands for the sum over all possible light neutrino production processes.
A runcard that can be used to reproduce these values can be found in appendix A. Beams were set to a center of mass energy of 13 TeV to be high enough to construct the heavy neutrinos.
Table 4.1 shows the comparison of Sherpa calculations to expectations. All widths are to be understood as the sum of all processes matching the given form including antiparticles. For

$m(N_1)$	Channel(s)	Explicit theory [GeV]	Sherpa [GeV]
0.1	$e^{\pm}\mu^{\mp}\overset{(-)}{\nu_e}$	$4.636 \cdot 10^{-19}$	$(4.628 \pm 0.010) \cdot 10^{-19}$
0.1	$\overset{(-)}{\nu_e} e^+ e^-$	$2.730 \cdot 10^{-19}$	$(2.73 \pm 0.06) \cdot 10^{-19}$
0.1	$\overset{(-)}{\nu_e} \nu_l \bar{\nu}_l$	$4.636 \cdot 10^{-19}$	$(4.62 \pm 0.06) \cdot 10^{-19}$
1	$e^{\pm}\mu^{\mp}\overset{(-)}{\nu_e}$	$4.636 \cdot 10^{-14}$	$(4.626 \pm 0.010) \cdot 10^{-14}$
1	$\overset{(-)}{\nu_e} l^+ l^-$	$3.312 \cdot 10^{-14}$	$(3.302 \pm 0.006) \cdot 10^{-14}$
1	$\overset{(-)}{\nu_e} \nu_l \bar{\nu}_l$	$4.636 \cdot 10^{-14}$	$(4.628 \pm 0.006) \cdot 10^{-14}$
5	$e^{\pm} l^{\mp}\overset{(-)}{\nu_l}$	$2.8976 \cdot 10^{-10}$	$(2.898 \pm 0.005) \cdot 10^{-10}$
5	$\overset{(-)}{\nu_e} l^+ l^-$	$1.2169 \cdot 10^{-10}$	$(1.2219 \pm 0.0018) \cdot 10^{-10}$
5	$\overset{(-)}{\nu_e} \nu_l \bar{\nu}_l$	$1.4488 \cdot 10^{-10}$	$(1.4519 \pm 0.0020) \cdot 10^{-10}$
100	Total width	0.2198	0.22098
100	$W^{\pm}e^{\mp}$	0.09764	0.097996
100	$Z^0\overset{(-)}{\nu_e}$	0.02485	0.024985
500	Total width	159.04	159.91
500	$W^{\pm}e^{\mp}$	41.009	41.234
500	$Z^0\overset{(-)}{\nu_e}$	40.955	41.180
500	$H\overset{(-)}{\nu_e}$	36.067	36.265
1000	$W^{\pm}e^{\mp}$	657.339	660.944
1000	$Z^0\overset{(-)}{\nu_e}$	328.642	330.444
1000	$H\overset{(-)}{\nu_e}$	318.420	320.166

Table 4.1: Comparison between partial and total width computed with Serpa and explicit predictions

$m_N > m_W$ the statistical error of Sherpa's cross section calculation is zero because those $1 \to 2$ decays are easier calculable elementary processes. The uncertainty of the theoretical widths is not given explicitly. The used formulas are the leading terms in mass expansion and thus the deviation is of the next order in mass. The results for $m_N = 1000$GeV are consistent with the theoretical high mass limes. Every channel out of Z, $W^{\pm}$ and H takes 25% branching fraction because at high masses only the mass of N matters for the width of each channel and all four widths become approximately equal. As Sherpa's calculation agrees well with the analytical predictions, the model implementation can be consideres validated.

4.3 Displacement Results

The displacement of high-energy decay vertices has successfully been implemented. Figure 4.4 shows the simulated lifetimes of Majorana neutrinos assuming $m_N = 1$GeV in their rest frame. An exponential decay law with the average lifetime $\tau = \frac{1}{\Gamma}$ is expected. One can see that the curve follows the exponential decay law of the red line which uses the theoretical decay width

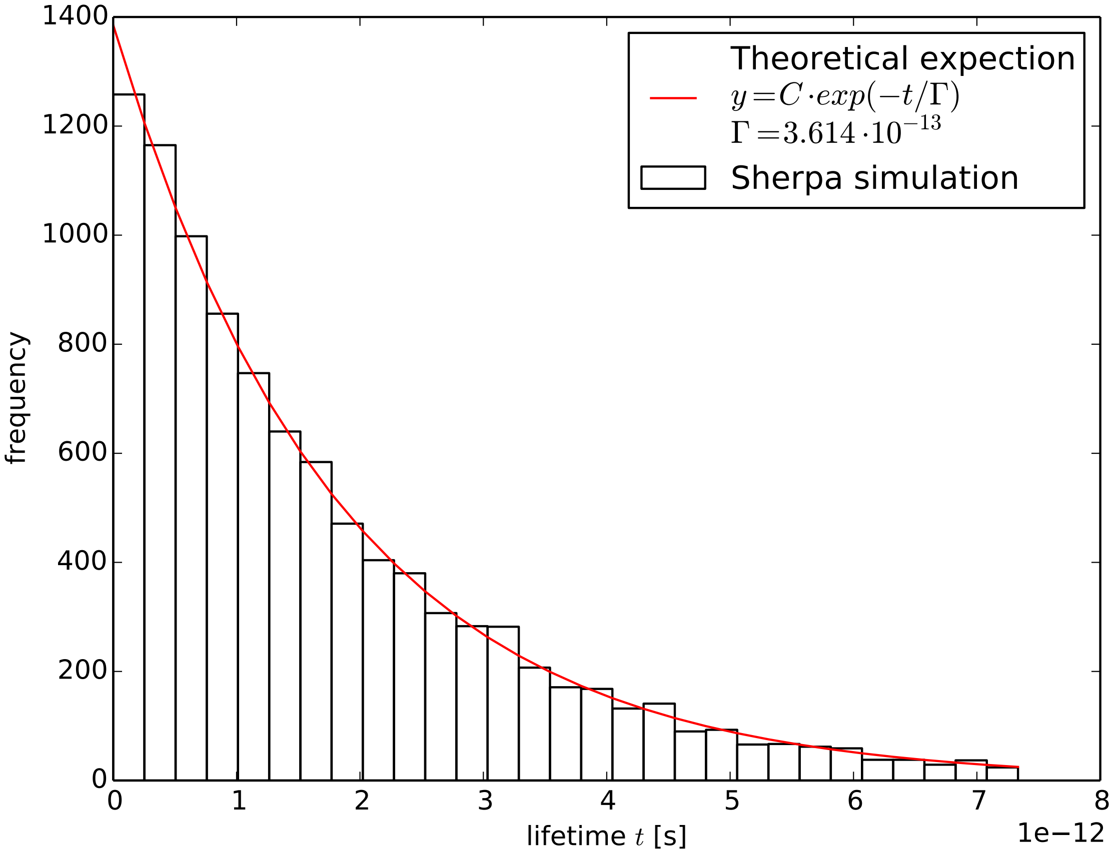

Figure 4.4: Distribution of N_1 unboosted lifetimes within 10000 simulated events at 13TeV with $m(N_1) = 1.0\text{GeV}$

$\Gamma = 3.614 \cdot 10^{-13}\text{GeV}$. So the average lifetime is expected to be

$$\tau_{\text{theory}} = \frac{1}{\Gamma} = \frac{1}{3.614 \cdot 10^{-13}\text{GeV}} = 2.767 \cdot 10^{12}\text{GeV} = 1.821 \cdot 10^{-12}\text{s} \ .$$

Indeed, the average simulated lifetime is in this case $\tau_{\text{sim}} = (1.862 \pm 0.018) \cdot 10^{-12}\text{s}$, which was determined directly out of the simulation data. Taking into account the statistical uncertainty, this result is convincing that the simulated decaying behavior fulfills the expactations.

Because of every decaying heavy neutrino having a finite lifetime and typically a nonzero momentum, the distance between its production and decay position is expected to be larger than zero. Figure 4.5 depicts the frequencies of runlengths for Majorana neutrinos with $m(N_1) = 1.0\text{GeV}$ with simulated production at a center-of-mass energy of 13TeV. The lengths are given in the laboratory system. While at first view the curve seems to have identical properties compared to Figure 4.4, it does not follow any exponential law, but in this case an exponential law is not expected because the lengths depend on velocities and are boosted into the laboratory system with varying boosts. This way, displacements depend on the energy scale given by the experiment. For this example 13TeV were used as center of mass

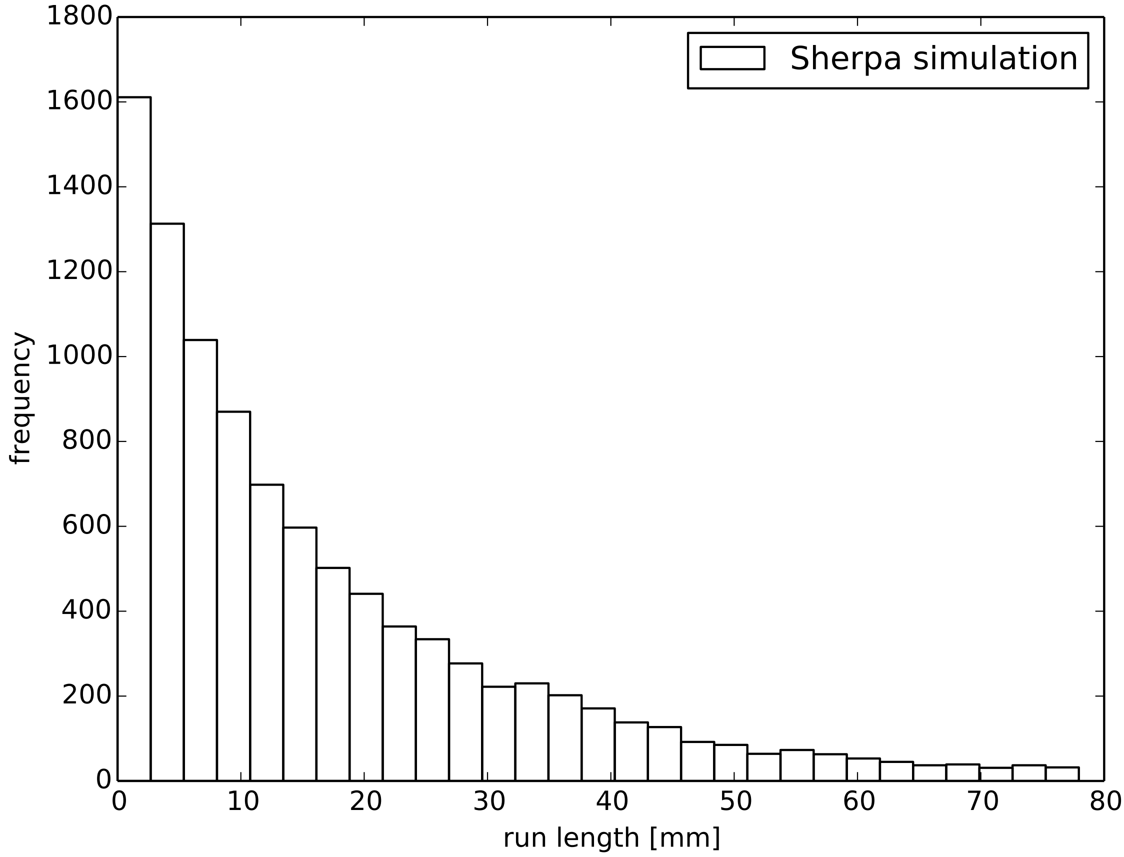

Figure 4.5: Distribution of N_1 run lengths in the laboratory system within 10000 simulated events at 13TeV with $m(N_1) = 1.0$GeV

energy, which is a typical setup at the LHC. The average propagated distance for this setup is (18.017 ± 0.20)mm. So if Majorana neutrinos are existing around the considered mass, one will propably detect them with these vertex offsets in experiment.

5 Summary and Outlook

5.1 Summary

The seesaw model, which is meant to explain how neutrinos have tiny but nonzero masses, predicts heavy right-handed Majorana neutrinos. One hopes to find those in high energy physics experiments with displaced vertices which can be simulated using Sherpa.
Sherpa has successfully been installed and the Seesaw model type I was included. Several features of the model have been tested including center-of-mass energies of Majorana decay products each for $1 \rightarrow 2$ and $1 \rightarrow 3$ decays. This gives the expected distribution having a peak at the Majorana neutrino's invariant mass with its expected decay width. Partial and total decay widths of heavy neutrino decays calculated with Sherpa were compared to theoretical predictions. This was done at different assumed masses lower as well as larger than m_W. The difference between the widths calculated directly form theory formulas and widths estimated with Sherpa is so small that Sherpa is assumed to work the right way.
Vertex displacements for elementary particle decays at a high energy scale were implemented. For that a new function has been implemented into Sherpa's source code. In this function lifetimes are generated using pseudo-random numbers according to an exponential decay law with the decay constant calculated out of the particle's decay width. The lifetime is boosted into the laboratory system. Using this boosted lifetime and the decying particle's momentum the vertex displacement is estimated. It is shown that the decaying behavior of Majorana neutrinos corresponds to the expected exponential decay law. Thus vertex displacements have successfully been implemented into Sherpa.

5.2 Outlook

In a Sherpa simulation with all phases activated the products of high energy decays are parton showered afterwards. A possible further task is to carry the displacements through the showering process. Then the positions of final particles will be comparable to experimental data.
For a further validation of Sherpa and the model one can compare further decay widths. For $m_N < m_W$ either the partial decay widths for hadronic decays or for elementary decays into quarks can be compared to explicitly calculated theoretical widths. Furthermore Majorana neutrino production processes can be validated. E.g. simulated production cross sections could

be compared to theoretical expectations.
On the experimental side further studies are necessary to possibly detect heavy neutrinos using displaced vertices. This way the limits on neutral heavy leptons could be extended.

A Example Run Card

The following is the content of a run card named “run.dat” which can be used for the simulation of selected production and decay of the Majorana neutrino N_1. All three Majorana neutrinos are set unstable. In the ufo section Majorana masses are set to $m(N_1) = 100\text{GeV}$, $m(N_2) = 300\text{GeV}$ and $m(N_3) = 500\text{GeV}$, the heavy neutrino mixing is set all diagnal. In the processes section the production process is set to $p_1 p_2 \to N_1 \nu_e$, where $p_{1,2}$ stand for arbitrary partons. In the run section the Majorana neutrino is forced to decay via the process $N_1 \to Z\nu_e$. By setting both beam energies to 6500 GeV the center of mass energy of the collision is set to the LHC of 13 TeV.

```
(run){
  # general settings
  EVENTS 1;

  # decay settings
  HARD_DECAYS=1;
  STABLE[9900012]=0
  STABLE[9900014]=0
  STABLE[9900016]=0
  HDH_STATUS[9900012,23,12]=2

  # matrix element generator setup
  ME_SIGNAL_GENERATOR Comix;
  SCALES VAR{Abs2(p[0]+p[1])};

  # model setup
  MODEL Seesaw_TypeI_UFO;

  # LHC beam setup:
  BEAM_1 2212; BEAM_ENERGY_1 6500;
  BEAM_2 2212; BEAM_ENERGY_2 6500;
}(run)
```

```
(processes){
  Process 93 93 -> 9900012 -12;
  End process;
}(processes)

(selector){
  # phase space cuts for matrix elements
}(selector)

(ufo){
block mass
23 91.1876
6 172
25 125.6
9900012 100.0
9900014 300.0
9900016 500.0
block sminputs
1 127.944
2 1.174741e-05
3 0.1184
block yukawa
6 172
15 1.777
block numixing
1 1.0
2 0.0
3 0.0
4 0.0
5 1.0
6 0.0
7 0.0
8 0.0
9 1.0
decay 23 2.4952
decay 24 2.085
decay 6 1.50833649
decay 25 0.00575308848
```

```
decay 9900012 0.303
decay 9900014 1.5
decay 9900016 12.3
}(ufo)
```

B Vertex Displacement Code

The following is the implementation of vertex displacements for high energy decays.

```
void Hard_Decay_Handler::SetPosition( ATOOLS::Blob* blob )
{
  Particle* inpart = blob->InParticle( 0 );

  // boost lifetime into lab
  double gamma = 1. / rpa->gen.Accu();
  if ( inpart->Flav().HadMass() > rpa->gen.Accu() ) // if m_0=0 within Accuracy
  {
    gamma = inpart->E() / inpart->Flav().HadMass(); // gamma = E/m
  }
  else { // For (quasi-) massless particles:
    double q2  = dabs( inpart->Momentum().Abs2() );
    if ( q2 > rpa->gen.Accu() ) gamma = inpart->E() / sqrt(q2); // gamma = E/q
  }
  double lifetime_boosted = gamma * inpart->Time();
  Vec3D  spatial = inpart->Distance( lifetime_boosted );
  Vec4D     position = Vec4D( lifetime_boosted * rpa->c(), spatial );
  blob->SetPosition( inpart->XProd() + position ); // in mm
}
```

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Erklärung

Hiermit erkläre ich, dass ich diese Arbeit im Rahmen der Betreuung am Institut für Kern- und Teilchenphysik ohne unzulässige Hilfe Dritter verfasst und alle Quellen als solche gekennzeichnet habe.

Hans Hoffmann
Dresden, June 2016