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The Calculation of Threshold Corrections to Renormalization Group Running in the Exceptional Supersymmetric Standard Model

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Kurzdarstellung

Das eingeschränkte Exzeptionelle Supersymmetrische Standardmodell (cE$_6$SSM) ist eine supersymmetrische Eichtheorie, basierend auf einer durch Stringtheorie motivierten großen vereinheitlichten Eichgruppe E_6. In einer 2009 erschienen Veröffentlichung [1] wurden erste Massenspektren des cE$_6$SSM berechnet. Die darin dargestellte Rechenprozedur benutzt die Methode der effektiven Feldtheorie, um das cE$_6$SSM bei niedrigen Energien in das Standardmodell zu überführen. Jedoch werden bei dieser Prozedur Schwellenkorrekturen für die Modellparameter vernachlässigt, die auftreten, wenn vom cE$_6$SSM zum Standardmodell übergegangen wird.

In dieser Arbeit werden Einschleifenschwellenkorrekturen zu den Eichkopplungen im cE$_6$SSM berechnet. Die Ergebnisse werden in einen Teilchenspektrumgenerator eingebaut, um eine genauere Vorhersage für die Teilchenmassen im cE$_6$SSM zu erhalten.

Abstract

The constrained Exceptional Supersymmetric Standard Model (cE$_6$SSM) is a supersymmetric gauge theory based on a string-inspired grand unified gauge group E_6. In a recent publication [1] first mass spectra of the cE$_6$SSM were calculated. The presented calculation procedure uses the effective field theory approach to match the cE$_6$SSM to the Standard Model at low energies. However, this procedure misses one-loop threshold corrections to the model parameters when matching the Standard Model to the cE$_6$SSM.

In this thesis the calculation of one-loop threshold corrections to the gauge couplings in the cE$_6$SSM is presented. The results are incorporated into a particle spectrum generation program to obtain a more precise prediction of particle masses in the cE$_6$SSM.

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1 Introduction

The Standard Model of particle physics is currently the best tested theory which provides a description of all known particles and the strong and electroweak interaction. But due to the missing incorporation of gravity, dark matter and dark energy, the Standard Model is not a complete description of nature.

Theories with local supersymmetry can provide a unification of the strong, electroweak and gravitational interaction and are therefore interesting candidates for physics beyond the Standard Model. However, since exact supersymmetry has not been observed in nature, such theories must include supersymmetry breaking terms in the Lagrangian in order to be valid at low energies. A desirable feature of such extended theories would be gauge coupling unification at a high scale. This would allow one to embed all gauge interactions and matter particles into a simple grand unified gauge group.

Theories of this kind naturally arise in the framework of heterotic $E_8 \times E_8'$ string theory, which can in the strong coupling limit be described by eleven-dimensional supergravity (M-Theory). By various breaking mechanisms supersymmetric grand unified theories (GUTs) with an E_6 gauge symmetry and further family symmetries can arise. These also provide a hidden sector to generate soft supersymmetry breaking at low energies.

The Exceptional Supersymmetric Standard Model ($\mathrm{E_6SSM}$) is a supersymmetric gauge theory, inspired by E_6 GUT models. It provides gauge coupling unification and the embedding of all matter fields into fundamental representations of the E_6. Furthermore, the $\mathrm{E_6SSM}$ avoids problems, which appear in minimal supersymmetric extensions of the Standard Model (see Section 3.2.1).

In this thesis a constrained version of the $\mathrm{E_6SSM}$ is studied, which is referred to as constrained Exceptional Supersymmetric Standard Model ($\mathrm{cE_6SSM}$). It is defined by extra unification constraints on the soft supersymmetry breaking parameters at the unification scale. The $\mathrm{cE_6SSM}$ parameters, which are defined at high energies are connected to the low energy ones by renormalization group running.

At low energies the $\mathrm{cE_6SSM}$ must match the observed physics which is well modeled by the Standard Model. In order to achieve this all new $\mathrm{E_6SSM}$ particles which are not included in the Standard Model must be heavy. If this is the case, one can make use of the Appelquist–Carazzone decoupling theorem which states, that in case of a very split particle spectrum one can remove the heavy particles from the theory without spoiling the prediction of low-energy observables. It is therefore possible to integrate out all new $\mathrm{E_6SSM}$ particles and derive the Standard Model at low energies. This is called effective field theory approach where the $\mathrm{E_6SSM}$ is the full and the Standard Model is the effective theory.

In a recent publication [1] a detailed procedure for calculating $\mathrm{cE_6SSM}$ particle spectra via the effective field theory approach was presented. A particle spectrum generator, based on SOFTSUSY 2.0.5 was written and first cE_6SSM spectra were calculated. However, the used calculation procedure misses threshold corrections to the model parameters, which must be included when integrating out the heavy particles. This leads to an unphysical dependency of the calculated masses on the arbitrary matching scale. This dependency is a measure of the theoretical error on the predicted mass spectrum. For the studied parameter points in [1] the error obtained by scale variation is typically 10–50 % of the particle mass. It is expected that the proper incorporation of threshold corrections into the program can reduce these errors significantly.

The aim of this thesis is to calculate one-loop threshold corrections to renormalization group running of the gauge couplings for the matching of the E_6SSM to the Standard Model. The results are implemented into the cE_6SSM particle spectrum generator and mass spectra with improved precision are calculated.

This thesis is structured as follows. In the first two sections an overview of the Standard Model, the E_6SSM and the cE_6SSM is given. Afterwards the general procedure of calculating threshold corrections is described in detail and the threshold corrections to the gauge couplings are calculated. In the last section the results are implemented into the particle spectrum generator and more precise particle spectra are presented.

2 The Standard Model

The Standard Model of particle physics is a quantum field theory theory which describes all known particles and interactions of nature, except gravity. It has been extensively tested and is by today the most precise description of the electromagnetic, weak and strong interaction [2, 3].

2.1 Symmetries and particle content

The Standard Model is a gauge theory with the gauge group $SU(3)_\mathrm{c} \times SU(2)_\mathrm{L} \times U(1)_Y$ with subscripts c, L, Y referring to color, left chirality and weak hypercharge. All matter fields (leptons and quarks) are fermions with spin 1/2. Their left and right chiral components transform as irreducible representations $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ of the Poincaré group. The corresponding Lie algebra is

$$[P^\mu, P^\nu] = 0 \tag{2.1}$$

$$[P^\mu, J^{\rho\sigma}] = \mathrm{i}(g^{\mu\rho} P^\sigma - g^{\mu\sigma} P^\rho) \tag{2.2}$$

$$[J^{\mu\nu}, J^{\rho\sigma}] = \mathrm{i}(g^{\nu\rho} J^{\mu\sigma} - g^{\mu\rho} J^{\nu\sigma} + g^{\mu\sigma} J^{\nu\rho} - g^{\nu\sigma} J^{\mu\rho}) \; . \tag{2.3}$$

Under the gauge group $SU(2)_\mathrm{L}$ left chiral fermions f_L transform as doublets and right chiral ones f_R as singlets. The full $SU(2)_\mathrm{L} \times U(1)_Y$ electroweak gauge transformation reads

$$f_L(x) = \frac{1}{2}(1 - \gamma_5) f(x) \to \exp\left(-\mathrm{i} g_1 \alpha_1(x) \frac{Y}{2} - \mathrm{i} g_2 \vec{\alpha}_2(x) \cdot \frac{\vec{\tau}}{2}\right) f_L(x) \tag{2.4}$$

$$f_R(x) = \frac{1}{2}(1 + \gamma_5) f(x) \to \exp\left(-\mathrm{i} g_1 \alpha_1(x) \frac{Y}{2}\right) f_R(x), \tag{2.5}$$

where g_1 and g_2 are the gauge couplings of $U(1)_Y$ and $SU(2)_\mathrm{L}$ respectively and $\alpha_1(x)$, $\vec{\alpha}_2(x)$ are the corresponding gauge functions. Furthermore Y is the hypercharge operator and $\vec{\tau}/2$ are the generators of $SU(2)_\mathrm{L}$.

Concerning the local $SU(3)_\mathrm{c}$ gauge group the quark and lepton fields of the Standard Model transform as

$$q_{L,R}(x) \to \exp\left(-\mathrm{i} g_3 \alpha_3^a(x) \frac{\lambda^a}{2}\right) q_{L,R}(x) \tag{2.6}$$

$$\ell_{L,R}(x) \to \ell_{L,R}(x) \tag{2.7}$$

where g_3 is the $SU(3)_\mathrm{c}$ gauge coupling and $\alpha_3^a(x)$ are the corresponding gauge functions. The Gell-Mann matrices λ^a as well as the generators of $SU(2)_\mathrm{L}$ form an algebra with the commutators

$$\left[\frac{\tau^i}{2}, \frac{\tau^j}{2}\right] = \mathrm{i}\varepsilon_{ijk}\frac{\tau^k}{2}\,, \qquad \left[\frac{\lambda^a}{2}, \frac{\lambda^b}{2}\right] = \mathrm{i}f^{abc}\frac{\lambda^c}{2}\,. \tag{2.8}$$

A complete listing of all Standard Model matter particles and their quantum numbers can be found in Table 2.1 and Table 2.2.

Table 2.1: Matter particles of the Standard Model (generation $i = 1, 2, 3$). In the last column dim rep G means the dimension of the representation of the groups $SU(3)_\mathrm{c}$, $SU(2)_\mathrm{L}$ and the Hypercharge $Y/2$.

	Field	Color	Weak isospin $\tau_3/2$	Hypercharge $Y/2$	dim rep G
Leptons	$\ell_{iL} = (\nu_{iL}\ \ e_{iL})^T$	0	$\pm 1/2$	$-1/2$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
	e_{iR}	0	0	-1	$(\mathbf{1}, \mathbf{1}, -1)$
Quarks	$q_{iL} = (u_{iL}\ \ d_{iL})^T$	$1, 2, 3$	$\pm 1/2$	$1/6$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$
	u_{iR}	$1, 2, 3$	0	$2/3$	$(\mathbf{3}, \mathbf{1}, \frac{2}{3})$
	d_{iR}	$1, 2, 3$	0	$-1/3$	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})$

Table 2.2: Gauge bosons of the Standard Model. In the last column dim rep G means the dimension of the representation of the groups $SU(3)_\mathrm{c}$, $SU(2)_\mathrm{L}$ and the Hypercharge $Y/2$.

Field	Gauge group	Coupling	Generators	dim rep G
B_μ	$U(1)_Y$	g_1	$Y/2$	$(\mathbf{1}, \mathbf{1}, 0)$
$\vec{W}_\mu$	$SU(2)_\mathrm{L}$	g_2	$\vec{\tau}/2$	$(\mathbf{1}, \mathbf{3}, 0)$
A^a_μ	$SU(3)_\mathrm{c}$	g_3	T^a	$(\mathbf{8}, \mathbf{1}, 0)$

2.2 Lagrangian density

After imposing the local gauge symmetry transformations (2.4)–(2.7) on all matter fields, the $SU(3)_\mathrm{c} \times SU(2)_\mathrm{L} \times U(1)_Y$ gauge invariant Lagrangian density reads

$$\mathcal{L} = \mathcal{L}_\mathrm{matter} + \mathcal{L}_\mathrm{gauge} \tag{2.9}$$

$$\mathcal{L}_\mathrm{matter} = \bar{\ell}_{iL}\mathrm{i}\not{D}\ell_{iL} + \bar{e}_{iR}\mathrm{i}\not{D}e_{iR} + \bar{q}_{iL}\mathrm{i}\not{D}q_{iL} + \bar{u}_{iR}\mathrm{i}\not{D}u_{iR} + \bar{d}_{iR}\mathrm{i}\not{D}d_{iR} \tag{2.10}$$

$$\mathcal{L}_\mathrm{gauge} = -\frac{1}{4}F^{\mu\nu}_a F^a_{\mu\nu} - \frac{1}{4}\vec{W}^{\mu\nu}\cdot\vec{W}_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}\,, \tag{2.11}$$

where it is summed over the generation index $i = 1, \ldots, 3$. The covariant derivative

$$D_\mu = \partial_\mu + \mathrm{i}g_3\frac{\lambda^a}{2}A^a_\mu + \mathrm{i}g_2\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu + \mathrm{i}g_1\frac{Y}{2}B_\mu \tag{2.12}$$

ensures the local gauge invariance, assuming the newly-introduced gauge fields A^a_μ, $\vec{W}_\mu$ and B_μ transform as

$$A^a_\mu \to A^a_\mu + \partial_\mu \alpha^a_3 - g_3 f^{abc} A^b_\mu \alpha^c_3 \qquad (a,b,c = 1,\ldots,8) \tag{2.13}$$
$$\vec{W}_\mu \to \vec{W}_\mu + \partial_\mu \vec{\alpha}_2 - g_2 \vec{W}_\mu \times \vec{\alpha}_2 \tag{2.14}$$
$$B_\mu \to B_\mu + \partial_\mu \alpha_1 \ . \tag{2.15}$$

The field strength tensors appearing in (2.11) are given by

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_3 f^{abc} A^b_\mu A^c_\nu \tag{2.16}$$
$$\vec{W}_{\mu\nu} = \partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu - g_2 \vec{W}_\mu \times \vec{W}_\nu \tag{2.17}$$
$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \ . \tag{2.18}$$

2.3 Electroweak symmetry breaking

The imposed gauge symmetry in Section 2.1 forbids mass terms for all gauge bosons of the theory. However, the gauge bosons Z and $W^\pm$ of the weak interaction, which were first observed in 1983 [4, 5, 6], are massive. This fact indicates that the $SU(2)_\mathrm{L} \times U(1)_Y$ symmetry must be broken at the weak scale to allow the weak gauge bosons to gain masses [7, 8, 9]. Since the photon has to remain massless, the Lagrangian of the broken theory must still have an unbroken local $U(1)_\mathrm{em}$ gauge symmetry.

To break the electroweak symmetry, the so-called Higgs field ϕ is postulated, which transforms as a doublet under $SU(2)_\mathrm{L}$,

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{2.19}$$

and has $Y = 1$. The Lagrangian density of the Standard Model is then augmented by an additional gauge invariant Higgs field term

$$\mathcal{L}_\mathrm{Higgs} = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi) \ , \tag{2.20}$$

where $V(\phi)$ is the most general renormalizable and gauge invariant Higgs potential

$$V(\phi) = -\mu \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \qquad \mu, \lambda > 0 \ . \tag{2.21}$$

After the field ϕ gets a non vanishing vacuum expectation value (VEV)

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \ , \qquad v = \sqrt{\frac{\mu^2}{\lambda}} \ , \tag{2.22}$$

arising from minimizing $V(\phi)$, the vacuum is no longer invariant under $SU(2)_\mathrm{L} \times U(1)_Y$. Therefore this symmetry is spontaneously broken. Expanding the Higgs field around its vacuum expectation value

$$\phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left[v + h(x) + \mathrm{i}G^0\right] \end{pmatrix} \tag{2.23}$$

yields mass and mixing terms for the gauge bosons B_μ and $\vec{W}_\mu$. Here $h(x)$ is the physical Higgs field and G^0, $G^\pm$ are Goldstone bosons. The latter arise when a continuous symmetry is spontaneously broken [10, 11]. They are unphysical and can be removed from the theory by a gauge transformation.

When rotating to mass eigenstates, one can identify one massless and three massive electroweak gauge bosons

$$W^\pm_\mu = \frac{1}{\sqrt{2}}\left(W^1_\mu \mp \mathrm{i}W^2_\mu\right) \qquad m_W = \frac{g_2}{2}v = m_Z \cos\theta_W \tag{2.24}$$

$$Z_\mu = \cos\theta_W W^3_\mu - \sin\theta_W B_\mu \qquad m_Z = \frac{v}{2}\sqrt{g_1^2 + g_2^2} \tag{2.25}$$

$$A_\mu = \sin\theta_W W^3_\mu + \cos\theta_W B_\mu \qquad m_A = 0\ . \tag{2.26}$$

Since A_μ is massless, it can be identified with the photon. The mixing angle θ_W is related to the gauge couplings by

$$\sin\theta_W = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}, \qquad \cos\theta_W = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}\ . \tag{2.27}$$

In the following the abbreviations $c_W \equiv \cos\theta_W$ and $s_W \equiv \sin\theta_W$ are used. The generator of the electric charge Q of the remaining $U(1)_{\mathrm{em}}$ gauge symmetry is related to the generators of $SU(2)_{\mathrm{L}}$ and $U(1)_Y$ via the Klein–Nishina formula

$$Q = \frac{\tau_3}{2} + \frac{Y}{2}\ . \tag{2.28}$$

The corresponding electromagnetic gauge coupling e is obtained from

$$e = g_2 \sin\theta_W = g_1 \cos\theta_W\ . \tag{2.29}$$

The nonzero VEV of the Higgs field is also responsible for giving masses to the fermions of the Standard Model via Yukawa interactions. The corresponding Lagrangian density is

$$\mathcal{L}_{\mathrm{Yukawa}} = -f^{e*}_{ij}\overline{\ell_{iL}}\phi e_{jR} - f^{d*}_{ij}\overline{q_{iL}}\phi d_{jR} - f^{u*}_{ij}\overline{q_{iL}}\phi^C u_{jR} + \mathrm{h.\,c.}\ , \tag{2.30}$$

where $\phi^C = \mathrm{i}\tau_2\phi^*$ is the charge conjugated Higgs field and the f^k_{ij} are the Yukawa couplings. The fermion mass matrices are then given by

$$m^e_{ij} = \frac{v}{\sqrt{2}}f^{e*}_{ij}, \qquad m^d_{ij} = \frac{v}{\sqrt{2}}f^{d*}_{ij}, \qquad m^u_{ij} = \frac{v}{\sqrt{2}}f^{u*}_{ij}\ . \tag{2.31}$$

2.4 Quantization

In order to quantize the Standard Model, it is necessary to introduce gauge-fixing terms and Faddeev–Popov ghost fields u, $\bar{u}$ [12, 13]. The latter eliminate unphysical degrees of freedom, which originate from the invariance of the Lagrangian under gauge transformation. It is most convenient to choose the gauge such that the resulting Lagrangian is renormalizable and mixing terms between the gauge bosons and the Goldstone fields do not occur. One possible realization of this is the R_ξ gauge

$$\mathcal{L}_{R_\xi} = -\frac{1}{2\xi_A}\left(C^A\right)^2 - \frac{1}{2\xi_Z}\left(C^Z\right)^2 - \frac{1}{\xi_W}C^+C^- - \frac{1}{2\xi_3}\left(C_3^a\right)^2 \ , \tag{2.32}$$

where the gauge-fixing operators are given by

$$C^A = \partial^\mu A_\mu \tag{2.33}$$
$$C^Z = \partial^\mu Z_\mu - m_Z \xi_Z G^0 \tag{2.34}$$
$$C^\pm = \partial^\mu W_\mu^\pm \mp \mathrm{i} m_W \xi_W G^\pm \tag{2.35}$$
$$C_3^a = \partial^\mu A_\mu^a \ . \tag{2.36}$$

The related Faddeev–Popov ghost field Lagrangian can then be written as $(i, j, k \in \{A, Z, \pm\})$

$$\mathcal{L}_{\text{ghost}} = \mathcal{L}_{\text{ghost}}^{\text{EW}} + \mathcal{L}_{\text{ghost}}^{\text{QCD}} \tag{2.37}$$
$$\mathcal{L}_{\text{ghost}}^{\text{EW}} = -\int \mathrm{d}^4z \, \mathrm{d}^4y \, \bar{u}^i(x) \left(\frac{\delta C^i(x)}{\delta V_\nu^k(z)} \frac{\delta V_\nu^k(z)}{\delta \alpha^j(y)} + \frac{\delta C^i(x)}{\delta G^k(z)} \frac{\delta G^k(z)}{\delta \alpha^j(y)} \right) u^j(y) \tag{2.38}$$
$$\mathcal{L}_{\text{ghost}}^{\text{QCD}} = -\int \mathrm{d}^4z \, \mathrm{d}^4y \, \bar{u}^a(x) \left(\frac{\delta C^a(x)}{\delta A_\nu^c(z)} \frac{\delta A_\nu^c(z)}{\delta \alpha_3^b(y)} \right) u^b(y) \tag{2.39}$$

where $V^Z \equiv Z$, $V^A \equiv A$ and $V^\pm \equiv W^\pm$. $\delta V_\nu^k(z)/\delta\alpha^j(y)$ here means the variation of the field $V_\nu^k(z)$ under infinitesimal gauge transformation led by the gauge parameter $\alpha^j(y)$ in terms of the mass eigenstates (2.24)–(2.26). The gauge parameters α^j are defined by

$$\alpha^\pm = \alpha_2^1 \mp \mathrm{i}\alpha_2^2 \tag{2.40}$$
$$\alpha^A = c_W \alpha_1 - s_W \alpha_2^3 \tag{2.41}$$
$$\alpha^Z = s_W \alpha_1 + c_W \alpha_2^3 \ . \tag{2.42}$$

The complete Standard Model Lagrangian is the sum of all above-mentioned contributions

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{R_\xi} + \mathcal{L}_{\text{ghost}} \ . \tag{2.43}$$

3 The constrained Exceptional Supersymmetric Standard Model

3.1 Supersymmetry

3.1.1 Supersymmetry algebra

The introduction of supersymmetry is motivated by the wish for an extension of the Poincaré algebra (2.3) to gain a symmetry between bosons and fermions [14]. This symmetry has first been observed within the framework of string theory [15, 16, 17] and was later applied to quantum field theories [18, 19].

The Haag, Łopuszánnski, Sohnius theorem [20] states, that the only possibility to extend the Poincaré algebra in a non-trivial way, is to introduce N sets of fermionic generators $\{Q^i_A, \bar{Q}^{\dot{A}}_i\}_{i=1\ldots N}$, which transform as the $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ representations of the proper orthochronous Lorentz group. In general there can be up to $N = 8$ of these sets, but only $N = 1$ supersymmetric quantum field theories are suitable to describe chiral fields [21]. Therefore only $N = 1$ supersymmetry is considered here. The Poincaré algebra (2.3) is then extended by the following commutation relations for the fermionic generators

$$\{Q_A, \bar{Q}_{\dot{B}}\} = 2\sigma^\mu_{A\dot{B}} P_\mu \ , \tag{3.1}$$

$$\{\bar{Q}^{\dot{A}}, \bar{Q}^B\} = 2\bar{\sigma}^{\mu\dot{A}B} P_\mu \tag{3.2}$$

$$\{Q_A, Q_B\} = \{\bar{Q}^{\dot{A}}, \bar{Q}^{\dot{B}}\} = 0 \tag{3.3}$$

$$[Q_A, P_\mu] = [\bar{Q}^{\dot{A}}, P_\mu] = 0 \tag{3.4}$$

$$[J_{\mu\nu}, Q_A] = -(\sigma_{\mu\nu})_A{}^B Q_B \ , \tag{3.5}$$

$$[J_{\mu\nu}, \bar{Q}^{\dot{A}}] = -(\bar{\sigma}_{\mu\nu})^{\dot{A}}{}_{\dot{B}} \bar{Q}^{\dot{B}} \ . \tag{3.6}$$

The operators Q_A and $\bar{Q}^{\dot{A}}$ convert bosonic states into fermionic ones and vice versa. States, which transform as irreducible representations of the Super-Poincaré algebra form chiral multiplets. The simplest one for $P^2 \neq 0$ is the Wess-Zumino multiplet, which contains a two-component Weyl spinor and a complex scalar. Both can be seen to be superpartners of one another. Since P^2 commutes with both Q_A and $\bar{Q}^{\dot{A}}$, the Weyl fermion and the scalar have the same mass.

3.1.2 Superfields on superspace

In order to construct a supersymmetric field theory, it is convenient to introduce the so-called superspace, which is a product space of the Minkowski space and four Grassmann spaces. The superspace coordinates are denoted by $(x^\mu, \theta^A, \bar{\theta}_{\dot{A}})$, where x^μ is an element of the Minkowski space and θ^A, $\bar{\theta}_{\dot{A}}$ are spinorial Grassmann coordinate variables, which span the Grassmannean subspace of the superspace.

Functions on the superspace are called superfields. Due to the nilpotency of the Grassmann variables (see Appendix A.4.1), the superfields can be expanded into a final number of component fields

$$\begin{aligned}\mathcal{F}(z) \equiv \mathcal{F}(x,\theta,\bar{\theta}) &= f(x) + \sqrt{2}\theta\xi(x) + \sqrt{2}\bar{\theta}\bar{\chi}(x) + \theta\theta M(x) + \bar{\theta}\bar{\theta}N(x) \\ &\quad + \theta\sigma^\mu\bar{\theta}A_\mu(x) + \theta\theta\;\bar{\theta}\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\;\theta\zeta(x) + \frac{1}{2}\theta\theta\;\bar{\theta}\bar{\theta}D(x)\;.\end{aligned} \tag{3.7}$$

Here, the two-component notation, defined in Appendix A.4, is used. The fields $f(x)$, $M(x)$, $N(x)$ and $D(x)$ are scalars, $A_\mu(x)$ is a vector field, $\xi_A(x)$, $\zeta_A(x)$ are left-handed and $\bar{\chi}^{\dot{A}}(x)$, $\bar{\lambda}^{\dot{A}}(x)$ are right-handed Weyl spinor fields.

A global supersymmetry transformation on the superspace transforms the superspace coordinates as

$$(x^\mu, \theta, \bar{\theta}) \to (x^\mu - \mathrm{i}\theta\sigma^\mu\bar{\epsilon} + \mathrm{i}\epsilon\sigma^\mu\bar{\theta}, \theta + \epsilon, \bar{\theta} + \bar{\epsilon})\;. \tag{3.8}$$

From this, one can find a representation of the generators Q_A and $\bar{Q}^{\dot{A}}$ on the superspace

$$Q_A = -\mathrm{i}\left(\partial_A + \mathrm{i}\sigma^\mu_{A\dot{B}}\bar{\theta}^{\dot{B}}\partial_\mu\right)\;, \qquad \bar{Q}^{\dot{A}} = -\mathrm{i}\left(\bar{\partial}^{\dot{A}} + \mathrm{i}\theta^B\sigma^\mu_{B\dot{B}}\epsilon^{\dot{B}\dot{A}}\partial_\mu\right)\;. \tag{3.9}$$

The transformation of a general superfield under (3.8) in terms of Q_A and $\bar{Q}^{\dot{A}}$ is then given by

$$\mathcal{F} \to \mathcal{F} + \mathrm{i}(\epsilon Q + \bar{\epsilon}\bar{Q})\mathcal{F}\;. \tag{3.10}$$

When expanding the right-hand side of (3.10) one can see that the D component field of $\mathcal{F}$ transforms into it itself plus a total spacetime derivative. Supersymmetry invariant actions can then be constructed from these terms.

Left and right chiral superfields

In order to construct irreducible chiral multiplets, one defines superfields Φ and $\Phi^\dagger$ by

$$\bar{\mathcal{D}}_{\dot{A}}\Phi = 0\;, \qquad \mathcal{D}_A\Phi^\dagger = 0\;. \tag{3.11}$$

The fields Φ and $\Phi^\dagger$ are called left and right chiral superfields. The derivative $\mathcal{D}_A := \partial_A - \mathrm{i}\sigma^\mu_{A\dot{B}}\bar{\theta}^{\dot{B}}\partial_\mu$ is constructed such that it transforms covariant under (3.8). In terms of left and right chiral superspace coordinates $y^\mu = x^\mu - \mathrm{i}\theta\sigma^\mu\bar{\theta}$ and $\bar{y}^\mu = x^\mu + \mathrm{i}\theta\sigma^\mu\bar{\theta}$ the so defined chiral superfields can be decomposed into

$$\Phi(y,\theta) = \phi(y) + \sqrt{2}\theta\xi(y) + \theta\theta F(y) \tag{3.12}$$

$$\Phi^\dagger(\bar{y},\bar{\theta}) = \phi^*(\bar{y}) + \sqrt{2}\bar{\theta}\bar{\xi}(y) + \bar{\theta}\bar{\theta}F^*(\bar{y}) \; . \tag{3.13}$$

The Weyl spinor field $\xi(y)$ and the complex scalar field $\phi(y)$ can now be seen to be superpartners of one another, whereas $F(y)$ is an auxiliary field. An important feature is that the F component of a chiral superfield transforms into itself plus a total spacetime derivative. This enables one to construct a general supersymmetric Lagrangian in terms of chiral superfields

$$\mathcal{L} = \int \mathrm{d}^4\theta \; \Phi_i^\dagger \Phi_i + \int \mathrm{d}^2\theta \, \Big(\mathcal{W}(\Phi_i) + \mathrm{h.\,c.} \Big) \tag{3.14}$$

$$\mathcal{W}(\Phi_i) = a_i\Phi_i + \frac{1}{2} m_{ij}\Phi_i\Phi_j + \frac{1}{3!} f_{ijk}\Phi_i\Phi_j\Phi_k \; . \tag{3.15}$$

The function $\mathcal{W}(\Phi_i)$ is called superpotential. The invariance of $\mathcal{L}$ under supersymmetry transformations implies that $\mathcal{W}$ is an analytic function in the chiral superfields Φ_i, i. e., it does not contain any of the $\Phi_i^\dagger$.

Vector superfields

Vector superfields $V(x,\theta,\bar{\theta})$ are superfields which obey the reality condition $V^\dagger = V$. This implies that the components f, A_μ and D are real and $\bar{\chi} = \bar{\xi}$, $N = M^*$, $\zeta = \lambda$.

One can construct vector superfields from chiral superfields, e. g., if $\mathrm{i}\Lambda$ is a chiral superfield, then $(\mathrm{i}\Lambda - \mathrm{i}\Lambda^\dagger)$ is a vector superfield. This and the fact that the sum of vector superfields is again a vector superfield enables one to define an abelian supergauge transformation by

$$V \to V + \mathrm{i}\Lambda - \mathrm{i}\Lambda^\dagger \; . \tag{3.16}$$

The name supergauge transformation is justified, because the $A_\mu(x)$ component field of V transforms as under the classical gauge transformation $A_\mu \to A_\mu - 2\partial_\mu \Im\mathrm{m}\,\phi$. The vector superfield V takes the most simple form in the Wess-Zumino gauge

$$V(z) = \theta\sigma^\mu\bar{\theta}A_\mu(x) + \theta\theta\,\bar{\theta}\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\,\theta\lambda(x) + \frac{1}{2}\theta\theta\,\bar{\theta}\bar{\theta}D(x) \; , \tag{3.17}$$

where the components f, $\bar{\xi}$, M are gauged to zero. The component fields $\lambda(x)$ and $\bar{\lambda}(x)$ can be seen as superpartners to the real vector field $A_\mu(x)$, whereas $D(x)$ is an auxiliary field.

Supergauge invariant Lagrangian density

A supergauge transformation for a non-abelian gauge group is defined by

$$\Phi \to \exp\left(-\mathrm{i}\Lambda(z)\right)\Phi\ , \qquad \bar{\mathcal{D}}^{\dot{A}}\Lambda^a(z)T^a = 0 \tag{3.18}$$

$$\Phi^\dagger \to \Phi^\dagger \exp\left(+\mathrm{i}\Lambda^\dagger(z)\right)\ , \qquad \mathcal{D}_A\Lambda^{a\dagger}(z)T^a = 0 \tag{3.19}$$

where $\Lambda(z) \equiv 2g\Lambda^a(z)T^a$, $\Lambda^a(z)$ is a vector superfield and T^a are the generators of the gauge group. If one introduces gauge vector superfields $V^a(z)$, which transform as

$$\mathrm{e}^V \to \mathrm{e}^{-\mathrm{i}\Lambda^\dagger}\mathrm{e}^V\mathrm{e}^{\mathrm{i}\Lambda}\ , \qquad V \equiv 2gV^aT^a \tag{3.20}$$

$$\mathrm{e}^{-V} \to \mathrm{e}^{-\mathrm{i}\Lambda}\mathrm{e}^{-V}\mathrm{e}^{\mathrm{i}\Lambda^\dagger}\ , \tag{3.21}$$

then terms like $(\Phi^\dagger \mathrm{e}^V\Phi)$ are supergauge invariant. To construct a kinetic term for the gauge fields, one introduces the supersymmetric field strength tensor

$$W_A = -\frac{1}{4}\bar{\mathcal{D}}\bar{\mathcal{D}}\mathrm{e}^{-V}\mathcal{D}_A\mathrm{e}^V\ , \qquad W_A = 2gW_A^aT^a \tag{3.22}$$

$$\bar{W}^{\dot{A}} = -\frac{1}{4}\mathcal{D}\mathcal{D}\mathrm{e}^V\bar{\mathcal{D}}^{\dot{A}}\mathrm{e}^{-V}\ , \qquad \bar{W}^{\dot{A}} = 2g\bar{W}^{\dot{A}a}T^a\ . \tag{3.23}$$

From (3.20) and (3.21) one can see that the so defined field strength tensors transform under gauge transformation as

$$W_A \to \mathrm{e}^{-\mathrm{i}\Lambda}W_A\mathrm{e}^{\mathrm{i}\Lambda}\ , \qquad \bar{W}^{\dot{A}} \to \mathrm{e}^{-\mathrm{i}\Lambda^\dagger}\bar{W}^{\dot{A}}\mathrm{e}^{\mathrm{i}\Lambda^\dagger}\ , \tag{3.24}$$

which implies that terms of the form $\mathrm{Tr}[W^AW_A]$ and $\mathrm{Tr}[\bar{W}_{\dot{A}}\bar{W}^{\dot{A}}]$ are gauge invariant. The general supersymmetry and supergauge invariant Lagrangian can then be written as

$$\mathcal{L} = \frac{1}{16g^2C_r}\int \mathrm{d}^2\theta\ \mathrm{Tr}\left[W^AW_A + \bar{W}_{\dot{A}}\bar{W}^{\dot{A}}\right] + \int \mathrm{d}^4\theta\ \Phi^\dagger\mathrm{e}^V\Phi + \int \mathrm{d}^2\theta\left(\mathcal{W}(\Phi) + \mathrm{h.\,c.}\right), \tag{3.25}$$

where the superpotential $\mathcal{W}(\Phi)$ must be gauge invariant. The constant C_r is the representation invariant of the gauge group in the adjoint representation as defined in Appendix B.2. In terms of component fields Eq. (3.25) reads [22, 23]

$$\begin{aligned}\mathcal{L} = {} & \mathrm{i}\xi_i\sigma_\mu\Delta^{\dagger\mu}_{ij}\bar{\xi}_j + \left|\Delta^\mu_{ij}\phi_j\right|^2 - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a + \mathrm{i}\lambda^a\sigma^\mu\Delta_\mu\bar{\lambda}^a \\ & - \sqrt{2}g\left(\bar{\lambda}^a\bar{\xi}_iT^a_{ij}\phi_j + \mathrm{h.\,c.}\right) - V(\phi_i,\phi_j^*) - \left[\frac{1}{2}\xi_i\xi_j\left.\frac{\partial^2\mathcal{W}(\Phi)}{\partial\Phi_i\partial\Phi_j}\right|_{\theta=\bar{\theta}=0} + \mathrm{h.\,c.}\right]\ ,\end{aligned} \tag{3.26}$$

where the covariant derivative and the scalar potential are given by

$$\Delta^\mu_{ij} = \delta_{ij}\partial^\mu + \mathrm{i}gA^\mu_aT^a_{ij} \tag{3.27}$$

$$V(\phi_i, \phi_j^*) = F_i F_j^* + \frac{1}{2} D^a D^a \ . \tag{3.28}$$

The auxiliary fields F_i and D^a can be eliminated from $\mathcal{L}$ by using their equations of motion $\partial\mathcal{L}/\partial D^a = \partial\mathcal{L}/\partial F_i = \partial\mathcal{L}/\partial F_i^* = 0$. This yields

$$F_i = -\left.\frac{\partial \mathcal{W}^\dagger}{\partial \Phi_i^\dagger}\right|_{\theta=\bar{\theta}=0} , \qquad D^a = -g\phi_i^\dagger T_{ij}^a \phi_j \ . \tag{3.29}$$

3.2 Definition of the E$_6$SSM

3.2.1 Motivation

The Exceptional Supersymmetric Standard Model (E$_6$SSM) is a supersymmetric gauge theory, based on a grand unified gauge group E_6 [24]. It is inspired by the heterotic string theory based on $E_8 \times E_8'$, which can in the strong coupling limit be described by eleven-dimensional supergravity (M-Theory) [25]. In this limit the string scale can be of the order of the unification scale M_X. When compactifying the extra dimensions, the E_8 gets broken down to one of its subgroups, which might be an E_6. What remains is gravitational interaction of the matter representations of E_6 with E_8'. In this way the E_8' plays the role of a hidden sector that can lead to spontaneous breaking of local supersymmetry. At lower energies the E_8' and E_6 decouple, which leaves a set of soft supersymmetry breaking terms in the E_6 sector.

In order to construct a valid low energy theory it is assumed that the E_6 is broken at the GUT scale M_X via the Hosotani mechanism [26] down the rank–6 subgroup $SU(3)_\mathrm{c} \times SU(2)_\mathrm{L} \times U(1)_Y \times U(1)_\psi \times U(1)_\chi$. The anomaly-free E_6 subgroups $U(1)_\psi$ and $U(1)_\chi$ are defined by the breaking of E_6 via,

$$E_6 \to SO(10) \times U(1)_\psi \to SU(5) \times U(1)_\chi \times U(1)_\psi \ . \tag{3.30}$$

The $U(1)_\chi \times U(1)_\psi$ gauge symmetry is reduced further to an effective $U(1)'$ symmetry by the combination

$$U(1)' = U(1)_\chi \cos\theta + U(1)_\psi \sin\theta \ . \tag{3.31}$$

From a phenomenological point of view it is convenient to choose the angle θ such that a multiplet exists, which transforms as a singlet under $SU(5) \times U(1)'$. The right-handed neutrino can then be associated to this multiplet, which then does not couple to any other field. In order to achieve this one has to set $\tan\theta = \sqrt{15}$. The resulting $U(1)'$ group is then called $U(1)_N$.

The μ problem

Besides the motivation from string theory, a supersymmetric model based on an E_6 gauge group can solve the μ problem of the Minimal Supersymmetric Standard Model (MSSM) [27] in a natural way. The MSSM superpotential contains a term which is bilinear in the two MSSM Higgs doublets H_u, H_d with a dimensionful coupling μ

$$\mathcal{W}_{\text{MSSM}} = \mu(H_d \cdot H_u) + \cdots \tag{3.32}$$
$$= -\mu(\tilde{h}_d \cdot \tilde{h}_u) + \cdots \tag{3.33}$$

The fields $\tilde{h}_d$, $\tilde{h}_u$ in (3.33) are the fermionic components of H_d and H_u, respectively. Since μ is a superpotential parameter, one would expect that μ is of the order of the unification scale M_X. But the electroweak symmetry breaking (EWSB) in the MSSM leads to a relation between μ, the EWSB parameter $\tan\beta$ and the soft supersymmetry breaking parameters $m_{h_u}^2$, $m_{h_d}^2$ (see Sec. 3.2.4)

$$\mu^2 = \frac{m_{h_d}^2 - m_{h_u}^2 \tan^2\beta}{\tan^2\beta - 1} - \frac{1}{2} m_Z^2 \, . \tag{3.34}$$

Hence, μ must be of the order of the electroweak scale and at the same time of the order of the soft supersymmetry breaking scale, if there is not too much miraculous cancellation.

This μ problem is solved by models with an extra $U(1)'$ gauge symmetry, under which the two Higgs fields H_u, H_d carry different charges. Then the bilinear μ term is forbidden by the additional gauge symmetry, but it can be dynamically created by introducing another singlet field S which couples to the Higgs fields via

$$\mathcal{W}_{U(1)'} = \lambda S(H_d \cdot H_u) + \cdots \, . \tag{3.35}$$

When the $U(1)'$ symmetry is broken, the scalar component of S gets a vacuum expectation value

$$\langle s \rangle = \frac{s}{\sqrt{2}} \, , \tag{3.36}$$

which results in an effective μ term in the superpotential

$$\mathcal{W}_{U(1)'} = \lambda S(H_d \cdot H_u) + \cdots \tag{3.37}$$
$$= -\mu_{\text{eff}}(\tilde{h}_d \cdot \tilde{h}_u) + \cdots \, , \tag{3.38}$$

where $\mu_{\text{eff}} = \lambda s/\sqrt{2}$. Note, that in E_6 based models this whole setup can arise in a natural way, because when breaking the E_6, the resulting gauge group can contain several $U(1)$ gauge symmetries. In addition the fundamental representation of E_6 has a suitable field content in which the fields S, H_u, H_d fit in.

3.2.2 Particle content

The E_6SSM contains three families of matter particles, each corresponding to a fundamental $(\mathbf{27})_i$ representation of the E_6 group. In addition to these, the model is equipped with two Higgs-like doublets H', $\bar{H}'$ both originating from a $(\mathbf{27})$ and $(\overline{\mathbf{27}})'$ representation to ensure gauge coupling unification at a high scale M_X.

In order to compare the particle content of the E$_6$SSM to the Standard Model, the $(\mathbf{27})_i$ multiplets are decomposed under the $SU(5) \times U(1)_N$ subgroup

$$(\mathbf{27})_i \to (\mathbf{10}, 1)_i + (\bar{\mathbf{5}}, 2)_i + (\bar{\mathbf{5}}, -3)_i + (\mathbf{5}, -2)_i + (\mathbf{1}, 5)_i + (\mathbf{1}, 0)_i \ , \tag{3.39}$$

where the $SU(5)$ multiplets again decompose under the Standard Model gauge group $SU(3)_\mathrm{c} \times SU(2)_\mathrm{L} \times U(1)_Y$

$$(\mathbf{5})_i \to (\mathbf{3}, \mathbf{1}, -\tfrac{1}{3})_i + (\mathbf{1}, \mathbf{2}, \tfrac{1}{2})_i \tag{3.40}$$

$$(\bar{\mathbf{5}})_i \to (\bar{\mathbf{3}}, \mathbf{1}, \tfrac{1}{3})_i + (\mathbf{1}, \mathbf{2}, -\tfrac{1}{2})_i \tag{3.41}$$

$$(\mathbf{10})_i \to (\mathbf{3}, \mathbf{2}, \tfrac{1}{6})_i + (\bar{\mathbf{3}}, \mathbf{1}, -\tfrac{2}{3})_i + (\mathbf{1}, \mathbf{1}, 1)_i \ . \tag{3.42}$$

(In Appendix B.5 a more complete listing of representation decompositions of the E_6 and its subgroups can be found.) One can see from Eq. (3.39)–(3.42), that all Standard Model particles as well as their superpartners fit into the multiplets $(\mathbf{10}, 1)_i$ and $(\bar{\mathbf{5}}, 2)_i$. The $(\bar{\mathbf{5}}, -3)$ and $(\mathbf{5}, -2)$ representations contain Higgs-like doublets H_{1i}, H_{2i} and exotic colored matter fields X_i, $\bar{X}_i$. The remaining $SU(5)$ singlets $(\mathbf{1}, 0)_i$ and $(\mathbf{1}, 5)_i$ equate to right-handed neutrinos and fields S_i, respectively. A complete particle listing can be found in Table 3.1 and Table 3.2.

Table 3.1: Superfields of the E$_6$SSM (generation index $i = 1, 2, 3$) and their group multiplets, where SM $\equiv SU(3)_\mathrm{c} \times SU(2)_\mathrm{L} \times U(1)_Y$. For the abelian gauge groups $U(1)_Y$ and $U(1)_N$, the quantum numbers $Y/2$ and $N/2$ are listed, instead of the dimension of the representation.

Field	SM $\times U(1)_N$	$SU(5) \times U(1)_N$	E_6
$Q_i = (Q_{u_i} \;\; Q_{d_i})^T$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6}, 1)_i$	$(\mathbf{10}, 1)_i$	$(\mathbf{27})_i$
$\bar{U}_i$	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3}, 1)_i$		
$\bar{E}_i$	$(\mathbf{1}, \mathbf{1}, 1, 1)_i$		
$\bar{D}_i$	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3}, 2)_i$	$(\bar{\mathbf{5}}, 2)_i$	
$L_i = (L_{\nu_i} \;\; L_{e_i})^T$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2}, 2)_i$		
$\bar{X}_i$	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3}, -3)_i$	$(\bar{\mathbf{5}}, -3)_i$	
$H_{1i} = (H_{1i}^0 \;\; H_{1i}^-)^T$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2}, -3)_i$		
X_i	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3}, -2)_i$	$(\mathbf{5}, -2)_i$	
$H_{2i} = (H_{2i}^+ \;\; H_{2i}^0)^T$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2}, -2)_i$		
S_i	$(\mathbf{1}, \mathbf{1}, 0, 5)_i$	$(\mathbf{1}, 5)_i$	
$\bar{N}_i$	$(\mathbf{1}, \mathbf{1}, 0, 0)_i$	$(\mathbf{1}, 0)_i$	
$H' = (H'^0 \;\; H'^-)^T$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2}, 2)$	$\ni (\bar{\mathbf{5}}, 2)'$	$\ni (\mathbf{27})'$
$\bar{H}' = (H'^+ \;\; H'^0)^T$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2}, -2)$	$\ni (\mathbf{5}, -2)'$	$\ni (\overline{\mathbf{27}})'$

The gauge bosons and their superpartners (the gauginos) fit into the adjoint $(\mathbf{78})$ representation of the E_6, which can be decomposed under $SU(3)_\mathrm{c} \times SU(2)_\mathrm{L} \times U(1)_Y \times U(1)_N$ (see also Appendix B.5)

$$(\mathbf{78}) \to (\mathbf{8}, \mathbf{1}, 0, 0) + (\mathbf{1}, \mathbf{3}, 0, 0) + (\mathbf{1}, \mathbf{1}, 0, 0) + (\mathbf{1}, \mathbf{1}, 0, 0) + \cdots \tag{3.43}$$

Table 3.2: Superfields of the E_6SSM from the $(\mathbf{27})_i$ and $(\mathbf{78})$ representations (generation index $i = 1, 2, 3$) in terms of superfields and their component fields. The spinor fields ψ and their charge conjugate ψ^C are related by $\psi^C = C\bar{\psi}^T$ where $C = \mathrm{i}\gamma^2\gamma^0$.

Superfield	Component fields		
	Spin 0	Spin 1/2	Spin 1
$Q_i = (Q_{u_i} \;\; Q_{d_i})^T$	$\tilde{q}_{iL} = (\tilde{u}_{iL} \;\; \tilde{d}_{iL})^T$	$q_{iL} = (u_{iL} \;\; d_{iL})^T$	
$\bar{U}_i$	$\tilde{u}^*_{iR}$	u^C_{iR}	
$\bar{D}_i$	$\tilde{d}^*_{iR}$	d^C_{iR}	
$L_i = (L_{\nu_i} \;\; L_{e_i})^T$	$\tilde{\ell}_{iL} = (\tilde{\nu}_{iL} \;\; \tilde{e}_{iL})^T$	$\ell_{iL} = (\nu_{iL} \;\; e_{iL})^T$	
$\bar{E}_i$	$\tilde{e}^*_{iR}$	e^C_{iR}	
$\bar{N}_i$	$\tilde{\nu}^*_{iR}$	ν^C_{iR}	
X_i	$\tilde{x}_{iL}$	x_{iL}	
$\bar{X}_i$	$\tilde{x}^*_{iR}$	x^C_{iR}	
$H_{1i} = (H^0_{1i} \;\; H^-_{1i})^T$	$h_{1i} = (h^0_{1i} \;\; h^-_{1i})^T$	$\tilde{h}_{1iL} = (\tilde{h}^0_{1iL} \;\; \tilde{h}^-_{1iL})^T$	
$H_{2i} = (H^+_{2i} \;\; H^0_{2i})^T$	$h_{2i} = (h^+_{2i} \;\; h^0_{2i})^T$	$\tilde{h}_{2iL} = (\tilde{h}^+_{2iL} \;\; \tilde{h}^0_{2iL})^T$	
S_i	s_i	$\tilde{s}_i$	
V^a_g		$\tilde{g}^a$	A^a_μ
$\vec{V}^W$		$\vec{\tilde{\lambda}}$	$\vec{W}_\mu$
V^Y		$\tilde{\lambda}^Y_0$	B_μ
V^N		$\tilde{\lambda}^N_0$	Z'_μ

The gluons are associated to $(\mathbf{8}, \mathbf{1}, 0, 0)$. The $(\mathbf{1}, \mathbf{3}, 0, 0)$ multiplet contains the weak gauge bosons and the two $U(1)$ gauge fields belong to the two $(\mathbf{1}, \mathbf{1}, 0, 0)$ representations. Since the E_6 is broken at the GUT scale M_X, the other gauge bosons are assumed to have masses of the order of M_X and therefore could be integrated out. However, the E_6SSM is not a complete GUT model, therefore it does not account for these heavy GUT gauge bosons by definition.

3.2.3 Lagrangian density

Starting from the general non-abelian supergauge invariant Lagrangian (3.25), one can construct $\mathcal{L}_{\mathrm{E_6SSM}}$ which is invariant under the E_6 gauge transformation. However, since the E_6 invariance is broken at the GUT scale,

$$E_6 \to SU(3)_{\mathrm{c}} \times SU(2)_{\mathrm{L}} \times U(1)_Y \times U(1)_N \;, \tag{3.44}$$

one only considers the resulting subgroup for the construction.

The general E_6SSM Lagrangian can be split into a supersymmetric and a soft supersymmetry breaking part

$$\mathcal{L}_{\mathrm{E_6SSM}} = \mathcal{L}_{\mathrm{SUSY}} + \mathcal{L}_{\mathrm{soft}} \tag{3.45}$$

$$\mathcal{L}_{\text{SUSY}} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{matter}} \; , \tag{3.46}$$

where the pure gauge part $\mathcal{L}_{\text{gauge}}$ is constructed from the first term in Eq. (3.25) and the matter part $\mathcal{L}_{\text{matter}}$ comes from the second term in Eq. (3.25). Both are listed in Appendix B.7. The soft supersymmetry breaking part $\mathcal{L}_{\text{soft}}$ is discussed in Sec. 3.2.4.

The superpotential

The most general renormalizable E_6 invariant superpotential is given by the trace of the $(\mathbf{27})_i \times (\mathbf{27})_j \times (\mathbf{27})_k$ and $(\mathbf{27})_i \times (\mathbf{27})_j$ representations of E_6. Decomposing them in terms of the Standard Model and $U(1)_N$ gauge subgroups leaves us with the $SU(3)_{\text{c}} \times SU(2)_{\text{L}} \times U(1)_Y \times U(1)_N$ gauge invariant superpotential of the broken E_6.

However, this superpotential is phenomenologically not valid [24, 1], because baryon number violating terms exist, which would lead to rapid proton decay. Also unacceptable large contributions to non-diagonal flavour transitions can arise from additional Yukawa interactions. In order to conserve baryon number and avoid flavour changing neutral currents, an approximate Z_2^H symmetry is imposed, under which all chiral superfields transform as odd, except H_{13}, H_{23} and S_3. The remaining superpotential reads

$$\begin{aligned}\mathcal{W}_{\text{E}_6\text{SSM}} = &- f^e_{ij}(H_{13}\cdot L_i)\bar{E}_j - f^d_{ij}(H_{13}\cdot Q_i)\bar{D}_j - f^u_{ij}(Q_i\cdot H_{23})\bar{U}_j \\ &+ \frac{1}{2}M_{ij}\bar{N}_i\bar{N}_j + h^E_{4j}(H_{13}\cdot H')\bar{E}_j + h^N_{4j}(H_{23}\cdot H')\bar{N}_j + \mu'(H'\cdot\bar{H}') \\ &+ \lambda_i S_3(H_{1i}\cdot H_{2i}) + \kappa_i S_3 X_i\bar{X}_i + f_{\alpha\beta}S_\alpha(H_{13}\cdot H_{2\beta}) + \tilde{f}_{\alpha\beta}S_\alpha(H_{1\beta}\cdot H_{23}) \; ,\end{aligned} \tag{3.47}$$

where the dot product for $SU(2)$ doublets is defined by $A\cdot B := \epsilon_{DE}A^D B^E$ and it is summed over all generations $i,j = 1,\ldots,3$. One can see from Eq. (3.47) that the exotics X_i, $\bar{X}_i$ are stable, which is in contrast to experiment [28, 29, 30]. Therefore the Z_2^H symmetry can not be exact. In order to break the Z_2^H symmetry and at the same time avoid a rapid proton decay, another discrete symmetry must be added. This can be done in two ways: either a Z_2^L symmetry is implemented, under which all superfields except the leptons are even (Model I) or one imposes a Z_2^B symmetry, under which the exotic quarks and leptons are odd whereas the others remain even (Model II). The superpotential (3.47) is then enlarged by one of the following terms

$$\mathcal{W}_{\text{Model I}} = g^Q_{ijk}X_i(Q_j\cdot Q_k) + g^q_{ijk}\bar{X}_i\bar{D}_j\bar{U}_k \tag{3.48}$$

$$\mathcal{W}_{\text{Model II}} = g^N_{ijk}\bar{N}_i X_j\bar{U}_k + g^E_{ijk}\bar{E}_i X_j\bar{U}_k + g^D_{ijk}(Q_i\cdot L_j)\bar{X}_k \; . \tag{3.49}$$

In Model I the exotic quarks can decay into two quarks (they are diquarks) and in Model II they are leptoquarks since they decay into a lepton and a quark. In order to still be consistent with experimental data, the Yukawa couplings $g^{\{Q,q,N,E,D\}}_{ijk}$ are assumed to be small (less than 10^{-3}).

By electroweak symmetry breaking one wants to generate masses for the quarks, leptons and exotic fermions via the vacuum expectation value of the Higgs bosons.

The structure of the superpotential (3.47) is such that it is sufficient that only the scalar components of H_{13}, H_{23} and S_3 get a non-zero VEV. To ensure this, a certain hierarchy between the Yukawa couplings must exist

$$\kappa_i \sim \lambda_3 \gtrsim \lambda_{1,2} \gg f_{\alpha\beta}, \tilde{f}_{\alpha\beta}, h^E_{4j}, h^N_{4j} \; . \tag{3.50}$$

This hierarchical structure allows a simplification of the superpotential (3.47). Integrating out the right-handed neutrinos and the additional Higgs fields H', $\bar{H}'$, which are assumed to be very heavy, and keeping only the dominant terms, one arrives at

$$\begin{aligned} \mathcal{W}_{\mathrm{E_6SSM}} \approx &- f^e_{33}(H_{13}\cdot L_3)\bar{E}_3 - f^d_{33}(H_{13}\cdot Q_3)\bar{D}_3 - f^u_{33}(Q_3\cdot H_{23})\bar{U}_3 \\ &+ \lambda_i S_3(H_{1i}\cdot H_{2i}) + \kappa_i S_3 X_i \bar{X}_i \; . \end{aligned} \tag{3.51}$$

3.2.4 Soft supersymmetry breaking

In Section 3.1.1 it was stated that the mass operator P^2 commutes with all other generators of the Super-Poincaré algebra. This implies, that all component fields of a chiral and vector superfield have the same mass. Since no superpartners of the quarks, leptons and gauge bosons have been observed, supersymmetry can not be exact. In the E$_6$SSM supersymmetry breaking terms can appear in the Lagrangian due to decoupled gravitational interaction from the hidden sector E_8' [31]. These terms are such that quadratic divergences do not occur in loop corrections—therefore they are called ‘soft supersymmetry breaking’. In the most general form the soft supersymmetry breaking Lagrangian reads

$$\mathcal{L}_{\mathrm{soft}} = -\phi_i^*(m^2)_{ij}\phi_j + \left(\frac{1}{3!}\mathcal{A}_{ijk}\phi_i\phi_j\phi_k - \frac{1}{2}\mathcal{B}_{ij}\phi_i\phi_j + \mathcal{C}_i\phi_i - \frac{1}{2}M\tilde{\lambda}^a\tilde{\lambda}^a + \mathrm{h.\,c.}\right) \tag{3.52}$$

where ϕ is the scalar component of a chiral vector field and $\tilde{\lambda}^a$ is the fermionic component of a vector superfield. Applied to the E$_6$SSM the soft supersymmetry breaking Lagrangian reads

$$\begin{aligned} \mathcal{L}_{\mathrm{soft}} = &- m^2_{\tilde{q}_{iL}}|\tilde{q}_{iL}|^2 - m^2_{\tilde{u}_{iR}}|\tilde{u}_{iR}|^2 - m^2_{\tilde{d}_{iR}}|\tilde{d}_{iR}|^2 - m^2_{\tilde{\ell}_{iL}}|\tilde{\ell}_{iL}|^2 - m^2_{\tilde{e}_{iR}}|\tilde{e}_{iR}|^2 \\ &- m^2_{\tilde{x}_{iL}}|\tilde{x}_{iL}|^2 - m^2_{\tilde{x}_{iR}}|\tilde{x}_{iR}|^2 - m^2_{h_{1i}}|h_{1i}|^2 - m^2_{h_{2i}}|h_{2i}|^2 - m^2_{s_i}|s_i|^2 \\ &- m^2_{h'}|h'|^2 - m^2_{\bar{h}'}|\bar{h}'|^2 - \Big(B'\mu' h'\cdot\bar{h}' + \mathrm{h.\,c.}\Big) \\ &- \Big[\lambda_i A_{\lambda_i} s_3(h_{1i}\cdot h_{2i}) + \kappa_i A_{\kappa_i} s_3 \tilde{x}_{iL}\tilde{x}^*_{iR} + (f^e A^e)_{33}(h_{13}\cdot\tilde{\ell}_{3L})\tilde{e}^*_{3R} \\ &\quad + (f^d A^d)_{33}(h_{13}\cdot\tilde{q}_{3L})\tilde{d}^*_{3R} + (f^u A^u)_{33}(\tilde{q}_{3L}\cdot h_{23})\tilde{u}^*_{3R} + \mathrm{h.\,c.}\Big] \\ &- \frac{1}{2}\left(M_3\bar{\tilde{g}}\tilde{g} + M_2\bar{\vec{\tilde{\lambda}}}\vec{\tilde{\lambda}} + M_1\bar{\tilde{\lambda}}^Y_0\tilde{\lambda}^Y_0 + M_1'\bar{\tilde{\lambda}}^N_0\tilde{\lambda}^N_0 + \mathrm{h.\,c.}\right) \; . \end{aligned} \tag{3.53}$$

It is obvious from Eq. (3.53) that supersymmetry is broken, because the scalar components of the chiral superfields as well as the fermionic components of the vector superfields gain additional mass contributions from $\mathcal{L}_{\mathrm{soft}}$. Hence the theory is not invariant under supersymmetry operators that satisfy $[P^2, Q_A] = [P^2, \bar{Q}^{\dot{A}}] = 0$.

3.2.5 Electroweak symmetry breaking

In order to be consistent with low energy phenomenology, the gauge symmetry of the E_6SSM must be broken to

$$SU(3)_{\mathrm{c}} \times SU(2)_{\mathrm{L}} \times U(1)_Y \times U(1)_N \to SU(3)_{\mathrm{c}} \times U(1)_{\mathrm{em}} \tag{3.54}$$

This is, analogously to the Standard Model, done with the Higgs mechanism. One can see from the E_6SSM matter Lagrangian (B.41) as well as from the superpotential (3.51) that, in order to give masses to the gauge bosons $W^{\pm}$, Z, Z' and the chiral superfields, three Higgs fields must be given a non-zero vacuum expectation value (VEV). In the E_6SSM these are chosen to be the scalar components of the third generation Higgs superfields H_{13}, H_{23} and the singlet superfield S_3

$$\langle h_{13} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix} , \qquad \langle h_{23} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix} , \qquad \langle s_3 \rangle = \frac{s}{\sqrt{2}} . \tag{3.55}$$

The E_6SSM Higgs potential, as well as the electroweak symmetry breaking conditions are listed in Appendix B.8. The masses of the vector bosons are then given by

$$m_W = \frac{g_2}{2}\sqrt{v_1^2 + v_2^2} , \qquad m_Z = \frac{1}{2}\sqrt{g_1^2 + g_2^2}\sqrt{v_1^2 + v_2^2} , \qquad m_{Z'} = \frac{N_{S_i}}{2}\, g_N\, s \tag{3.56}$$

and the masses of the matter fermions are obtained from the Yukawa terms

$$m_{ij}^e = \frac{v_1}{\sqrt{2}} f_{ij}^{e*} , \qquad m_{ij}^d = \frac{v_1}{\sqrt{2}} f_{ij}^{d*} , \qquad m_{ij}^u = \frac{v_2}{\sqrt{2}} f_{ij}^{u*} , \qquad m_{ij}^x = \frac{s}{\sqrt{2}} \kappa_i \delta_{ij} . \tag{3.57}$$

From Eq. (3.56) one can identify the Standard Model Higgs VEV $v = \sqrt{v_1^2 + v_2^2}$. Furthermore it is convenient to introduce the abbreviation $\tan\beta := v_2/v_1$.

Z–Z' mixing

In a general E_6 based model the additional gauge boson Z' which corresponds to the $U(1)_N$ gauge symmetry can mix with the Z. However, for values of $s \gtrsim 1.5$ TeV this mixing can be neglected, which is done in the following.

Another important point to mention is, that due to loop effects at low energies the gauge groups $U(1)_Y$ and $U(1)_N$ can mix, which results in a mixing term for the gauge fields

$$\mathcal{L}_{\text{gauge,kin}} = -\frac{\sin\chi}{2} B^{\mu\nu} Z'_{\mu\nu} , \tag{3.58}$$

where $B_{\mu\nu}$ and $Z'_{\mu\nu}$ are the field strength tensors of the gauge bosons B_μ and Z'_μ respectively. The mixing parameter $\sin\chi$ equals zero at the GUT scale. However the mixing is small at low energies which is why it is neglected here.

3.2.6 Mass eigenstates

Due to soft supersymmetry breaking as well as electroweak symmetry breaking it happens that field mixing terms arise in the Lagrangian. They are removed by transforming all fields into mass eigenstates. Because of the different quantum numbers, several mixing sectors exist.

Mixing of squarks and sleptons

In general the mass terms of the mixing sfermions can be written in the form

$$\mathcal{L}_{\text{sfermion,mass}} = -\sum_{\tilde{\mathbf{f}}} \tilde{\mathbf{f}}^\dagger \mathcal{M}^2_{\tilde{\mathbf{f}}} \tilde{\mathbf{f}} \,, \tag{3.59}$$

where $\mathcal{M}^2_{\tilde{\mathbf{f}}}$ is the mixing matrix and $\tilde{\mathbf{f}}$ is a vector which contains all mixing sfermions for a certain mixing sector. In the E$_6$SSM one has in particular

$$\begin{aligned}
\text{up-type sleptons}: &\qquad \tilde{\mathbf{f}} = (\tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau)^T && (3.60)\\
\text{down-type sleptons}: &\qquad \tilde{\mathbf{f}} = (\tilde{e}_L, \tilde{\mu}_L, \tilde{\tau}_L, \tilde{e}_R, \tilde{\mu}_R, \tilde{\tau}_R)^T && (3.61)\\
\text{up-type squarks}: &\qquad \tilde{\mathbf{f}} = (\tilde{u}_L, \tilde{c}_L, \tilde{t}_L, \tilde{u}_R, \tilde{c}_R, \tilde{t}_R)^T && (3.62)\\
\text{down-type squarks}: &\qquad \tilde{\mathbf{f}} = (\tilde{d}_L, \tilde{s}_L, \tilde{b}_L, \tilde{d}_R, \tilde{s}_R, \tilde{b}_R)^T && (3.63)\\
\text{exotic squarks}: &\qquad \tilde{\mathbf{f}} = (\tilde{x}_{1L}, \tilde{x}_{2L}, \tilde{x}_{3L}, \tilde{x}_{1R}, \tilde{x}_{2R}, \tilde{x}_{3R})^T \,. && (3.64)
\end{aligned}$$

The mixing matrix $\mathcal{M}_{\tilde{\mathbf{f}}}$ is diagonalized by a unitary matrix $W^{\tilde{f}}$ in such a way that

$$\mathcal{L}_{\text{sfermion,mass}} = -\sum_{\tilde{\mathbf{f}}} \tilde{\mathbf{f}}^\dagger \mathcal{M}^2_{\tilde{\mathbf{f}}} \tilde{\mathbf{f}} = -\sum_{\tilde{\mathbf{f}}} \tilde{\mathbf{f}}^{m\dagger} M^2_{\tilde{\mathbf{f}}} \tilde{\mathbf{f}}^m \,, \tag{3.65}$$

where $M^2_{\tilde{\mathbf{f}}} = W^{\tilde{f}\dagger} \mathcal{M}^2_{\tilde{\mathbf{f}}} W^{\tilde{f}}$ is diagonal and $\tilde{\mathbf{f}}^m = W^{\tilde{f}\dagger}\tilde{\mathbf{f}}$ are mass eigenstates. In the following flavour mixing is neglected which implies that only left and right-handed sfermions of a certain type mix. The matrix $W^{\tilde{f}}$ can then be written as

$$W^{\tilde{f}}_{ii} = W^{\tilde{f}}_{i+3,i+3} = \cos\theta_{\tilde{f}_i} \,, \qquad (i = 1,2,3) \tag{3.66}$$

$$W^{\tilde{f}}_{i,i+3} = -W^{\tilde{f}}_{i+3,i} = -\sin\theta_{\tilde{f}_i} \,, \tag{3.67}$$

where i is the generation index and $\theta_{\tilde{f}_i}$ are the mixing angles. The mass eigenstates for a specific flavour are then labeled by numbers $1, 2$, thus

$$\begin{aligned}
\text{up-type sleptons}: &\qquad \tilde{\mathbf{f}}^m = (\tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau)^T && (3.68)\\
\text{down-type sleptons}: &\qquad \tilde{\mathbf{f}}^m = (\tilde{e}_1, \tilde{\mu}_1, \tilde{\tau}_1, \tilde{e}_2, \tilde{\mu}_2, \tilde{\tau}_2)^T && (3.69)\\
\text{up-type squarks}: &\qquad \tilde{\mathbf{f}}^m = (\tilde{u}_1, \tilde{c}_1, \tilde{t}_1, \tilde{u}_2, \tilde{c}_2, \tilde{t}_2)^T && (3.70)\\
\text{down-type squarks}: &\qquad \tilde{\mathbf{f}}^m = (\tilde{d}_1, \tilde{s}_1, \tilde{b}_1, \tilde{d}_2, \tilde{s}_2, \tilde{b}_2)^T && (3.71)\\
\text{exotic squarks}: &\qquad \tilde{\mathbf{f}}^m = (\tilde{x}_{11}, \tilde{x}_{21}, \tilde{x}_{31}, \tilde{x}_{12}, \tilde{x}_{22}, \tilde{x}_{32})^T \,. && (3.72)
\end{aligned}$$

Charginos and neutralinos

As mentioned above, the diverse symmetry breaking mechanisms also provide non-diagonal mass terms for the gauginos and the higgsinos. Therefore mixing into neutral and charged fermionic mass eigenstates occurs, which are called neutralinos and charginos.

The Lagrangian for the mixing charged fields after electroweak symmetry breaking can be written as

$$\mathcal{L}_{\text{chargino,mass}} = -(\psi^-)^T \mathbf{X} \psi^+ + \text{h.c.}\,, \tag{3.73}$$

where the two-column vectors ψ^+, ψ^- are constructed from the two-component Weyl spinors of the charged gauginos and higgsinos

$$\psi^+ = \begin{pmatrix} \lambda^+ \\ \tilde{h}_{23}^+ \end{pmatrix}\,, \qquad \psi^- = \begin{pmatrix} \lambda^- \\ \tilde{h}_{13}^- \end{pmatrix}\,, \qquad \tilde{\lambda}^\pm = \frac{1}{\sqrt{2}}\left(\tilde{\lambda}_1 \mp \mathrm{i}\tilde{\lambda}_2\right) \tag{3.74}$$

and

$$\mathbf{X} = \begin{pmatrix} M_2 & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & \mu_{\text{eff}} \end{pmatrix}\,. \tag{3.75}$$

The matrix $\mathbf{X}$ can be diagonalized by two unitary matrices U, V which then also define four two-component chargino mass eigenstates. The latter can be combined to two four-component Dirac spinors $\chi_{1,2}^\pm$ and the Lagrangian (3.73) becomes

$$\mathcal{L}_{\text{chargino,mass}} = -\sum_{i=1}^{2} m_{\chi_i^\pm} \overline{\chi_i^+} \chi_i^+ \tag{3.76}$$

where the chargino masses are

$$m^2_{\chi^\pm_{1,2}} = \frac{1}{2}\left[M_2^2 + \mu_{\text{eff}}^2 + 2m_W^2 \pm \sqrt{(M_2^2 + \mu_{\text{eff}}^2 + 2m_W^2)^2 - 4(M_2\mu_{\text{eff}} - m_W^2 \sin 2\beta)^2}\right]. \tag{3.77}$$

Analogous to the charginos, the Lagrangian of the mixing neutral gauginos and higgsinos can be written as

$$\mathcal{L}_{\text{neutralino,mass}} = -\frac{1}{2}(\psi^0)^T \mathbf{Y} \psi^0 + \text{h.c.}\,, \tag{3.78}$$

where the involved fields are collected in the vector

$$\psi^0 = (\tilde{\lambda}_0^Y, \tilde{\lambda}_3, \tilde{h}_{13}^0, \tilde{h}_{23}^0, \tilde{s}_3, \tilde{\lambda}_0^N)^T \tag{3.79}$$

and the mixing matrix $\mathbf{Y}$ reads

$$\mathbf{Y} = \begin{pmatrix} M_1 & 0 & -\frac{1}{2}g_1 v_1 & \frac{1}{2}g_1 v_2 & 0 & 0 \\ 0 & M_2 & \frac{1}{2}g_2 v_1 & -\frac{1}{2}g_2 v_2 & 0 & 0 \\ -\frac{1}{2}g_1 v_1 & \frac{1}{2}g_2 v_1 & 0 & -\mu_{\text{eff}} & -\frac{\lambda_3 v_2}{\sqrt{2}} & \frac{N_{H_{13}}}{2} g_N v_1 \\ \frac{1}{2}g_1 v_2 & -\frac{1}{2}g_2 v_2 & -\mu_{\text{eff}} & 0 & -\frac{\lambda_3 v_1}{\sqrt{2}} & \frac{N_{H_{23}}}{2} g_N v_2 \\ 0 & 0 & -\frac{\lambda_3 v_2}{\sqrt{2}} & -\frac{\lambda_3 v_1}{\sqrt{2}} & 0 & \frac{N_S}{2} g_N s \\ 0 & 0 & \frac{N_{H_{13}}}{2} g_N v_1 & \frac{N_{H_{23}}}{2} g_N v_2 & \frac{N_S}{2} g_N s & M_1' \end{pmatrix} . \tag{3.80}$$

The appearing quantum numbers $N/2$ of the Higgs fields are listed in Table 3.1. When $\mathbf{Y}$ is diagonalized the six resulting Weyl spinor mass eigenstates are combined to six Majorana spinorial fields χ_i^0 $(i = 1, \dots, 6)$ such that the Lagrangian (3.78) can be written as

$$\mathcal{L}_{\text{neutralino,mass}} = -\frac{1}{2} \sum_{i=1}^{6} m_{\chi_i^0} \overline{\chi_i^0} \chi_i^0 . \tag{3.81}$$

Higgs sector

Analogous to the charginos and neutralinos, the charged Higgs bosons do not mix with the neutral ones, because of charge conservation. It is also assumed that CP is conserved, which implies that the neutral Higgses mix to CP even and CP odd mass eigenstates. The used E_6SSM Higgs potential is listed in Appendix B.8.

In the charged sector, the bosons $h_{13}^{\pm}$ and $h_{23}^{\pm}$ mix to massive Higgs states $H^{\pm}$ and massless Goldstone bosons $G^{\pm}$ via

$$\begin{pmatrix} G^{\pm} \\ H^{\pm} \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} h_{13}^{\pm} \\ h_{23}^{\pm} \end{pmatrix} \tag{3.82}$$

with masses†

$$m_{H^{\pm}}^2 = \frac{\sqrt{2}\lambda_3 A_{\lambda_3}}{\sin 2\beta} s - \frac{\lambda_3^2}{2} v^2 + \frac{g_2^2}{4} v^2 \tag{3.83}$$

$$m_{G^{\pm}}^2 = 0 . \tag{3.84}$$

The Higgs fields with negative charge are defined by $h_{13}^- := (h_{13}^+)^\dagger$ and $h_{23}^- := (h_{23}^+)^\dagger$.

From the imaginary parts of the neutral Higgses and from the imaginary part of the singlet field, neutral mass eigenstates are composed, which transform odd under charge and parity transformation CP. Two of them are Goldstone bosons and one is a massive pseudoscalar Higgs called A. They are constructed via

$$A = P_S \sin\varphi + P \cos\varphi \qquad (CP \text{ odd Higgs}) \tag{3.85}$$

$$G' = P_S \cos\varphi - P \sin\varphi \tag{3.86}$$

†In [1] Eq. (47) for the masses of $H^{\pm}$ misses an additional factor $1/2$ in the term $\sim g_2^2$. This factor must be present, because in the supersymmetric MSSM limit it must be $m_{H^{\pm}}^2 \to \frac{g_2^2}{4} v^2 = m_W^2$.

$$G^0 = \sqrt{2}(\Im\mathrm{m}\, h^0_{13} \cos\beta - \Im\mathrm{m}\, h^0_{23} \sin\beta)\ , \tag{3.87}$$

where the fields P, P_S are given by

$$P = \sqrt{2}(\Im\mathrm{m}\, h^0_{13} \sin\beta + \Im\mathrm{m}\, h^0_{23} \cos\beta) \tag{3.88}$$

$$P_S = \sqrt{2}\, \Im\mathrm{m}\, s_3 \tag{3.89}$$

and the mixing angle φ can be expressed by the vacuum expectation values

$$\tan\varphi = \frac{v}{2s} \sin 2\beta\ . \tag{3.90}$$

Their masses are

$$m^2_A = \frac{\sqrt{2}\lambda_3 A_{\lambda_3}}{\sin 2\varphi} v \tag{3.91}$$

$$m^2_{G^0} = m^2_{G'} = 0\ . \tag{3.92}$$

From the real parts of h^0_{13}, h^0_{23} and s_3 one can construct three CP even Higgs bosons. In the field basis $(\mathsf{h}_1, \mathsf{h}_2, \mathsf{h}_3)$ their mass matrix takes the simple form

$$m^2_{\Re\mathrm{e}\, h^0} = \begin{pmatrix} \frac{\partial^2 V}{\partial v^2} & \frac{1}{v}\frac{\partial^2 V}{\partial v \partial\beta} & \frac{\partial^2 V}{\partial v \partial s} \\ \frac{1}{v}\frac{\partial^2 V}{\partial v \partial\beta} & \frac{1}{v^2}\frac{\partial^2 V}{\partial^2 \beta} & \frac{1}{v}\frac{\partial^2 V}{\partial s \partial\beta} \\ \frac{\partial^2 V}{\partial v \partial s} & \frac{1}{v}\frac{\partial^2 V}{\partial s \partial\beta} & \frac{\partial^2 V}{\partial^2 s} \end{pmatrix}\ , \tag{3.93}$$

where the h_i are defined by the rotation

$$\Re\mathrm{e}\, h^0_{13} = (\mathsf{h}_1 \cos\beta - \mathsf{h}_2 \sin\beta + v_1)/\sqrt{2} \tag{3.94}$$

$$\Re\mathrm{e}\, h^0_{23} = (\mathsf{h}_1 \sin\beta + \mathsf{h}_2 \cos\beta + v_2)/\sqrt{2} \tag{3.95}$$

$$\Re\mathrm{e}\, s_3 = (\mathsf{h}_3 + s)/\sqrt{2}\ . \tag{3.96}$$

For practical reasons $m^2_{\Re\mathrm{e}\, h^0}$ is diagonalized numerically in explicit calculations. Its mass eigenstates are labeled h_1, h_2, h_3. The explicit form of the components of the mass matrix (3.93) can be found in [24].

3.2.7 The constrained E$_6$SSM

The soft supersymmetry breaking Lagrangian (3.53) is responsible for most of the parameters of the E$_6$SSM. In addition to the 19 Standard Model ones, the E$_6$SSM contains further 148 real parameters [1]. To reduce their number, a constrained model is considered, which is motivated by minimal supergravity. As stated in Sec. 3.2.1 the breakdown of the heterotic string theory symmetry group $E_8 \times E'_8$ down to $E_6 \times E'_8$ can result in weak gravitational interaction of the matter representations of E_6 with the E'_8. The condensation of gauginos in the hidden sector E'_8 can then lead to soft supersymmetry breaking terms in the Lagrangian of the observable sector [32, 33, 34]. In this framework all scalar and gaugino masses as well as the trilinear couplings can be universal at the unification scale. These universality conditions can be written as

$$m_i^2(M_X) = m_0^2 \tag{3.97}$$
$$M_i(M_X) = M_{1/2} \tag{3.98}$$
$$A_i(M_X) = A_0 \ , \tag{3.99}$$

where m_0 is the universal scalar mass, $M_{1/2}$ is the universal gaugino mass and A_0 is the universal trilinear coupling. Furthermore one assumes gauge coupling unification at the GUT scale

$$g_0 = g_3(M_X) = g_2(M_X) = \sqrt{\frac{5}{3}}\, g_1(M_X) = \sqrt{40}\, g_N(M_X) \ . \tag{3.100}$$

The resulting model is referred to as the "constrained Exceptional Supersymmetric Standard Model" (cE$_6$SSM) which has only nine free parameters

$$\lambda_i(M_X), \kappa_i(M_X), m_0^2, M_{1/2}, A_0 \ . \tag{3.101}$$

By using the three electroweak symmetry breaking conditions

$$\frac{\partial V}{\partial v_1} = \frac{\partial V}{\partial v_2} = \frac{\partial V}{\partial s} = 0 \tag{3.102}$$

one can replace m_0^2, $M_{1/2}$ and A_0 by the EWSB parameters v, s and $\tan\beta$. Since v can be taken from experiment [3] eight free parameters remain for the cE$_6$SSM

$$\lambda_i(M_X), \kappa_i(M_X), s, \tan\beta \ . \tag{3.103}$$

4 Calculation of threshold corrections in the E_6SSM

In this chapter the reader is introduced to the idea of effective field theories and the matching of an effective to a full theory. Afterwards the general procedure of calculating threshold corrections in described and applied to the matching of the E_6SSM to the Standard Model.

4.1 Effective field theories

In order to describe physical phenomena at a certain energy scale E_1, it is often not needed to know about the underlying theory, which is valid up to much higher energies $E_2 \gg E_1$. Instead it is sufficient to use a so-called effective theory at the scale E_1, which might be constructed from the more complete one in the limit $E_1 \ll E_2$. The advantage of using an effective theory is often that it simplifies the calculation of observables. However, it turns out that when going from a more fundamental theory to an effective one, the resulting theory is in general not renormalizable. This fact can change the view of current non-renormalizable theories like for example general relativity, which can then be seen as an approximation of a more fundamental prescription.

4.1.1 The Appelquist–Carazzone decoupling theorem

In 1974 Appelquist and Carazzone showed [35], that starting from a theory with widely separated mass scales one can construct an effective field theory, which coincides with the complete theory in the limit of low momenta. The theorem is proved in a physical (on-shell) renormalization scheme and states that the effect of heavy particles with mass M to low-energy scattering amplitudes with light external fields is suppressed by powers of p^2/M^2. This makes the heavy particles decouple in the limit $p^2 \ll M^2$ and a valid effective theory can be constructed, which does not include the heavy particles.

The question arises whether the decoupling theorem holds for other (unphysical) renormalization schemes. In general this can be proved by showing that the renormalization scheme dependence can be compensated by finite counter-terms (threshold corrections) [36]. In particular it was proved in the path integral formalism that the decoupling theorem still holds for abelian and non-abelian gauge theories if both the full and the effective theory are renormalized in the minimal subtraction scheme [37, 38].

4.1.2 The path integral point of view

A systematic way of constructing an effective theory from a given field theory with a spread mass spectrum within the path integral formalism was first proposed by Weinberg [39] and Ovrut and Schnitzer [40]. The idea is to construct the action of the effective theory by performing a functional integration over the heavy fields of the full action. In order to make this procedure more concrete the construction is examined within a toy model.

4.1.3 A toy model

Consider a theory with two fields, one heavy (H) and one light (l). The corresponding Lagrangian reads

$$\mathcal{L}^{\text{full}}(l,H) = \frac{1}{2}(\partial l)^2 - \frac{m^2}{2}l^2 + \frac{1}{2}(\partial H)^2 - \frac{M^2}{2}H^2 + \frac{g}{3!}l^3 + \frac{\lambda}{2!}lH^2 \,, \tag{4.1}$$

where $m^2 \ll M^2$. The theory is assumed to be $\overline{\text{MS}}$ renormalized, so that all divergences appearing in loop graphs are removed [41, 42]. The aim is now to construct an effective Lagrangian $\mathcal{L}^{\text{eff}}$ which only depends on the light field and which describes the same observables in the limit $p^2 \ll M^2$.

The generating functional for the 1PI correlation functions of the full theory reads (see Appendix A.3)

$$\Gamma^{\text{full}}[l_c,H_c] = -\mathrm{i}\log N \int \mathcal{D}l\,\mathcal{D}H\;\exp\left(\mathrm{i}\int \mathrm{d}^4x\,\left[\mathcal{L}^{\text{full}}(l+l_c,H+H_c) + JH + jl\right]\right) . \tag{4.2}$$

The corresponding correlation function for the effective theory Γ^{eff} is now obtained by the requirement that no heavy particles shall appear in initial or finals states. This implies that

$$\frac{\delta\Gamma^{\text{eff}}}{\delta H_c} = 0 \qquad \text{and} \qquad \frac{\delta W^{\text{eff}}}{\delta J} = 0 \,. \tag{4.3}$$

(The definition of $W[J]$ is given in Appendix A.3.) To achieve this one sets

$$\Gamma^{\text{eff}}[l_c] := \Gamma^{\text{full}}[l_c,0]\Big|_{J=0} \tag{4.4}$$

$$= -\mathrm{i}\log N \int \mathcal{D}l\,\mathcal{D}H\;\exp\left(\mathrm{i}\int \mathrm{d}^4x\,\left[\mathcal{L}^{\text{full}}(l+l_c,H) + jl\right]\right) . \tag{4.5}$$

The action of the effective theory is now obtained by performing the integration over $\mathcal{D}H$ in Eq. (4.5), which yields

$$\Gamma^{\text{eff}}[l_c] = -\mathrm{i}\log N \int \mathcal{D}l\;\exp\left(\mathrm{i}\int \mathrm{d}^4x\,\left[\mathcal{L}^{\text{eff}}(l+l_c) + jl\right]\right) , \tag{4.6}$$

where $\mathcal{L}^{\text{eff}}$ is defined by

$$\exp\left(\mathrm{i}\int \mathrm{d}^4x\ \mathcal{L}^{\text{eff}}(l+l_c)\right) = \int \mathcal{D}H\ \exp\left(\mathrm{i}\int \mathrm{d}^4x\ \mathcal{L}^{\text{full}}(l+l_c,H)\right). \tag{4.7}$$

To calculate $\mathcal{L}^{\text{eff}}(l)$ one splits the full Lagrangian into a part which depends only on l and a rest

$$\mathcal{L}^{\text{full}}(l,H) = \mathcal{L}_1^{\text{full}}(l) + \mathcal{L}_2^{\text{full}}(l,H) \tag{4.8}$$

$$\mathcal{L}_1^{\text{full}}(l) = \frac{1}{2}(\partial l)^2 - \frac{m^2}{2}l^2 + \frac{g}{3!}l^3 \tag{4.9}$$

$$\mathcal{L}_2^{\text{full}}(l,H) = \frac{1}{2}(\partial H)^2 - \frac{M^2}{2}H^2 + \frac{\lambda}{2!}lH^2 \equiv \frac{1}{2}H\Gamma_{HH}H \tag{4.10}$$

where

$$\Gamma_{HH} = \Big[-\Box - M^2 + \lambda l\Big]. \tag{4.11}$$

Performing the integral in Eq. (4.7) now yields the effective action

$$\int \mathrm{d}^4x\ \mathcal{L}^{\text{eff}}(l+l_c) = \int \mathrm{d}^4x\ \mathcal{L}_1^{\text{full}}(l+l_c) + \frac{\mathrm{i}}{2}\log\det\Gamma_{HH}\,. \tag{4.12}$$

One can see, that the effective Lagrangian is given by the part of the full Lagrangian, which contains only the light field, plus a correction term. To see what this last term means, one uses

$$\det\Gamma_{HH} = \exp\Big(\operatorname{Tr}\log\Gamma_{HH}\Big) \tag{4.13}$$

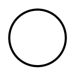
which yields

$$\frac{\mathrm{i}}{2}\log\det\Gamma_{HH} = \frac{\mathrm{i}}{2}\operatorname{Tr}\log\left[(-\Box - M^2)\left(1+\frac{\lambda l(x)}{-\Box - M^2}\right)\right] \tag{4.14}$$

$$= \frac{\mathrm{i}}{2}\operatorname{Tr}\log(-\Box - M^2) + \frac{\mathrm{i}}{2}\sum_{n=1}^{\infty} -\frac{1}{n}\operatorname{Tr}\left(\frac{\mathrm{i}}{-\Box - M^2}(\mathrm{i}\lambda)l(x)\right)^n. \tag{4.15}$$

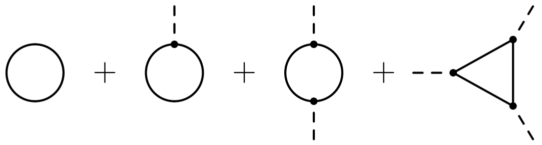
All terms in Eq. (4.15) have the form of one-loop graphs with heavy particles within the loop. The first one corresponds to an infinite sum of vacuum diagrams which can be absorbed into the normalization of the path integral. It is represented diagrammatically by a one-loop diagram

$$\frac{\mathrm{i}}{2}\operatorname{Tr}\log(-\Box - M^2) = \frac{1}{2} \tag{4.16}$$



where heavy particles are drawn with a solid line and light particles with a dashed line. The sum in Eq. (4.15) contains non-local one-loop contributions to the action of the effective theory. The n-th summand represents a n-point function with light external fields and heavy fields within the loop. Thus symbolically written

$$\frac{\mathrm{i}}{2}\log\det\Gamma_{HH} = \frac{1}{2}$$



$$+\cdots \tag{4.17}$$

where prefactors are omitted. In the following the various one-loop diagrams from the last term in Eq. (4.15) are calculated to obtain the effective action.

$n = 1$ tadpole contribution

The term with $n = 1$ is a tadpole contribution to the action as can be seen from writing out the trace

$$-\frac{\mathrm{i}}{2}\,\mathrm{Tr}\left(\frac{\mathrm{i}}{-\Box - M^2}(\mathrm{i}\lambda)l(x)\right) = -\frac{\mathrm{i}}{2}\int \mathrm{d}^4x\,(\mathrm{i}\lambda)l(x)S_F(0) \tag{4.18}$$

$$= \int \mathrm{d}^4x\, l(x)\left(-\frac{\lambda}{2(4\pi)^2}A_0(M)\right), \tag{4.19}$$

where $S_F(x-y)$ is the Feynman propagator in position space for a heavy field with mass M. $A_0(M)$ is a one-loop integral evaluated in dimensional regularization [41]

$$\frac{\mathrm{i}}{(4\pi)^2}\,A_0(M) = \mu^{4-D}\int\frac{\mathrm{d}^Dk}{(2\pi)^D}\frac{1}{k^2 - M^2 + \mathrm{i}\epsilon} \tag{4.20}$$

$$= \frac{\mathrm{i}}{(4\pi)^2}\,M^2\left(\Delta - \log\frac{M^2}{\mu^2} + 1\right) + \mathcal{O}(\epsilon) \tag{4.21}$$

where $D = 4 - 2\epsilon$ and $\Delta = \frac{1}{\epsilon} - \gamma_\mathrm{E} + \log 4\pi$.

$n = 2$ quadratic contribution

The summand $n = 2$ is purely non-local

$$-\frac{\mathrm{i}}{4}\,\mathrm{Tr}\left(\frac{\mathrm{i}}{-\Box - M^2}(\mathrm{i}\lambda)l(x)\right)^2$$

$$= -\frac{\mathrm{i}}{4}\int \mathrm{d}^4x_1\,\mathrm{d}^4x_2\,(\mathrm{i}\lambda)^2 l(x_1)S_F(x_2 - x_1)l(x_2)S_F(x_1 - x_2)\,. \tag{4.22}$$

However, in the limit $p^2 \ll M^2$ it becomes local, because then the Feynman propagator becomes diagonal

$$S_F(x-y) = \int\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{\mathrm{i}}{p^2 - M^2 + \mathrm{i}\epsilon}\,\mathrm{e}^{-\mathrm{i}p(x-y)} \tag{4.23}$$

$$= -\frac{\mathrm{i}}{M^2}\left(1 - \frac{\Box}{M^2} + \cdots\right)\delta(x-y)\ . \tag{4.24}$$

This means, that in this limit one can write Eq. (4.22) as a local action

$$\begin{aligned} -\frac{\mathrm{i}}{4}\,\mathrm{Tr}\left(\frac{\mathrm{i}}{-\Box - M^2}(\mathrm{i}\lambda)l(x)\right)^2 \\ = \int \mathrm{d}^4x\ \left[\frac{1}{2}(\partial l)^2\left(\frac{\lambda^2}{2(4\pi)^2}\frac{1}{6M^2}\right) - \frac{m^2}{2}l^2\left(\frac{\lambda^2}{2(4\pi)^2}\frac{1}{m^2}\log\frac{M^2}{\mu^2}\right)\right], \end{aligned} \tag{4.25}$$

where terms of the order $\mathcal{O}(p^4/M^4)$ are neglected to avoid non-renormalizable operators with dimension $d > 4$. The fact that in the limit $p^2 \ll M^2$ the one-loop contributions become local means that a construction of an effective field theory is possible in this limit.

$n = 3$ cubic contribution

An analogous calculation yields for $n = 3$

$$-\frac{\mathrm{i}}{6}\,\mathrm{Tr}\left(\frac{\mathrm{i}}{-\Box - M^2}(\mathrm{i}\lambda)l(x)\right)^3 = \int \mathrm{d}^4x\ \frac{g}{3!}l^3(x)\left(\frac{\lambda^3/g}{2(4\pi)^2 6M^2}\right). \tag{4.26}$$

$n = 4$ quartic contribution

Finally the calculation for $n = 4$ yields

$$-\frac{\mathrm{i}}{8}\,\mathrm{Tr}\left(\frac{\mathrm{i}}{-\Box - M^2}(\mathrm{i}\lambda)l(x)\right)^4 = \int \mathrm{d}^4x\ \frac{1}{4!}l^4(x)\left(\frac{\lambda^4}{4!(4\pi)^2 6M^4}\right). \tag{4.27}$$

The effective Lagrangian

Collecting all one-loop contributions to the effective action leads to the Lagrangian for the effective theory

$$\begin{aligned} \mathcal{L}^{\mathrm{eff}}(l) = \frac{1}{2}(\partial l)^2\left[1 + \frac{\lambda^2}{2(4\pi)^2 6M^2}\right] - \frac{m^2}{2}l^2\left[1 + \frac{\lambda^2}{2(4\pi)^2 m^2}\log\frac{M^2}{\mu^2}\right] \\ + \frac{g}{3!}l^3\left[1 + \frac{\lambda^3/g}{2(4\pi)^2 6M^2}\right] + \frac{1}{4!}l^4\left[\frac{\lambda^4}{4!(4\pi)^2 6M^4}\right], \end{aligned} \tag{4.28}$$

where tadpole terms and higher-dimensional operators are omitted. An interesting point here is that integrating out the heavy field H leads not only to corrections to the l^2 and l^3 terms, but also generates an new interaction term $\sim l^4$ which is suppressed by $1/M^4$.

When neglecting the l^4 term, the effective Lagrangian looks like the one of the full theory where all terms involving heavy fields are left out and the fields and parameters are renormalized. More precisely the effective Lagrangian can then be written as

$$\mathcal{L}^{\text{eff}}(\hat{l}) = \frac{1}{2}\left(\partial\hat{l}\right)^2 - \frac{\hat{m}^2}{2}\hat{l}^2 + \frac{\hat{g}}{3!}\hat{l}^3 \,, \tag{4.29}$$

if the renormalization transformations for $\hat{l}$, $\hat{g}$ and $\hat{m}$ read

$$\hat{l} = l\left(1 + \frac{1}{2}K_l\right) \,, \qquad K_l = \frac{\lambda^2}{2(4\pi)^2 6M^2} \tag{4.30}$$

$$\hat{g} = g + \delta g \,, \qquad \delta g = K_l \; g\left(\frac{\lambda}{g} - \frac{3}{2}\right) \tag{4.31}$$

$$\hat{m}^2 = m^2 + \delta m^2 \,, \qquad \delta m^2 = -K_l \; m^2 + \frac{\lambda^2}{2(4\pi)^2}\log\frac{M^2}{\mu^2} \,. \tag{4.32}$$

The terms δg and δm^2 are called threshold corrections. They describe the effect of the heavy particles on the parameters of the effective theory. This is in particular important for the renormalization group equations, where the beta functions in the effective theory do not contain contributions from heavy particles. The inclusion of threshold corrections accounts for these particles.

The 1PI correlation functions

Having constructed the effective field theory like above, it now follows that all one-loop 1PI correlation functions, which have light fields l at external lines, and some of their derivatives are equal at external momentum $p = 0$, more precisely

$$\Gamma^{\text{full}}_{ll}(p)\Big|_{p^2=0} = \Gamma^{\text{eff}}_{ll}(p)\Big|_{p^2=0} \tag{4.33}$$

$$\partial_{p^2}\,\Gamma^{\text{full}}_{ll}(p)\Big|_{p^2=0} = \partial_{p^2}\,\Gamma^{\text{eff}}_{ll}(p)\Big|_{p^2=0} \tag{4.34}$$

$$\Gamma^{\text{full}}_{lll}(p)\Big|_{p^2=0} = \Gamma^{\text{eff}}_{lll}(p)\Big|_{p^2=0} \,. \tag{4.35}$$

That means, that both the effective and the full theory describe the same physics in the limit $p \to 0$. The reason why not all derivatives of all 1PI correlation functions are equal at $p = 0$ is that one stops the expansion in powers of p^2/M^2 if operators of dimension $d > 4$ arise. In a renormalizable effective theory, only the equality of the first derivatives of the 1PI two-point functions holds in addition to equality of the 1PI correlation functions itself.

One can now reverse the argument. Suppose one is given a renormalizable full field theory and an associated renormalizable effective theory written in the form of Eq. (4.29). A way to determine the relations between the parameters as in Eq. (4.30)–(4.32) is to equate all necessary 1PI correlation functions and the first derivatives of the 1PI two-point functions in both theories at $p = 0$. This approach is called "matching" and will be used in the following.

4.2 General matching procedure

Consider a renormalizable field theory with the Lagrangian

$$\mathcal{L}^{\text{full}} = \mathcal{L}^{\text{full}}(\rho_1, \dots, \rho_p; l_1, \dots, l_q, H_1, \dots, H_r) \;, \tag{4.36}$$

which contains fields l_i with light masses, fields H_i with heavy masses and p parameters ρ_i. Integrating out the heavy fields $H_1, \dots, H_r$ results in an effective theory with a Lagrangian

$$\mathcal{L}^{\text{eff}} = \mathcal{L}^{\text{eff}}(\hat{\rho}_1, \dots; \hat{l}_1, \dots, \hat{l}_q) \;, \tag{4.37}$$

which now depends on light effective fields $\hat{l}_i$ and in general arbitrary many effective parameters $\hat{\rho}_i$.

In general $\mathcal{L}^{\text{eff}}$ is non-renormalizable, because it contains arbitrary many operator products with mass dimension $d > 4$. As shown in the previous section, these operators are suppressed by powers of $1/m_{H_i}^n$ where m_{H_i} are the masses of the heavy fields H_i. Neglecting these unrenormalizable operators in the limit $p \to 0$, one arrives at a renormalizable effective Lagrangian with a finite number of effective parameters.

In order to obtain a relation between the parameters ρ_i of the full and the $\hat{\rho}_i$ of the effective theory, one demands the following matching condition: All renormalized 1PI correlation functions with light external fields and the first derivatives of the renormalized 1PI two-point functions shall be equal at zero external momenta

$$\Gamma^{\text{full}}_{l_{i_1} l_{i_2} \cdots}(\rho_1, \dots, \rho_p)\Big|_{k_i=0} = \Gamma^{\text{eff}}_{l_{i_1} l_{i_2} \cdots}(\hat{\rho}_1, \dots, \hat{\rho}_k)\Big|_{k_i=0} \tag{4.38}$$

$$\partial^n_{k_i} \Gamma^{\text{full}}_{l_{i_1} l_{i_2}}(\rho_1, \dots, \rho_p)\Big|_{k_i=0} = \partial^n_{k_i} \Gamma^{\text{eff}}_{l_{i_1} l_{i_2}}(\hat{\rho}_1, \dots, \hat{\rho}_k)\Big|_{k_i=0} \qquad (n \leq 2) \;. \tag{4.39}$$

This implies that both theories describe the same observables in the limit $k_i \to 0$.

The next step is to decompose the renormalized 1PI correlation functions into a tree-level part and a part which contains one-loop contributions. In case of Γ^{full}, the loop part can again be split into a term, which contains only light fields $l_1, \dots, l_q$ in the loop and a rest, which contains heavy fields $H_1, \dots, H_r$

$$\Gamma^{\text{full}}_{l_{i_1} l_{i_2} \cdots} = \Gamma^{\text{full,tree}}_{l_{i_1} l_{i_2} \cdots} + \Gamma^{\text{full,1L,light}}_{l_{i_1} l_{i_2} \cdots} + \Gamma^{\text{full,1L,heavy}}_{l_{i_1} l_{i_2} \cdots} \tag{4.40}$$

$$\Gamma^{\text{eff}}_{l_{i_1} l_{i_2} \cdots} = \Gamma^{\text{eff,tree}}_{l_{i_1} l_{i_2} \cdots} + \Gamma^{\text{eff,1L,light}}_{l_{i_1} l_{i_2} \cdots} \;. \tag{4.41}$$

Imposing a relative field renormalization for the renormalized fields in the full and effective theory

$$\hat{l}_i = \left(1 + \frac{1}{2} K_{l_i}\right) l_i \;, \qquad (i = 1, \dots, q) \tag{4.42}$$

and inserting Eq. (4.40)–(4.42) into (4.38) yields

$$\Gamma^{\text{full,tree}}_{l_{i_1} l_{i_2} \cdots} + \Gamma^{\text{full,1L,light}}_{l_{i_1} l_{i_2} \cdots} + \Gamma^{\text{full,1L,heavy}}_{l_{i_1} l_{i_2} \cdots}$$

$$= \left(1 + \frac{1}{2}\sum_n K_{l_{i_n}}\right)\Gamma^{\text{eff,tree}}_{\hat{l}_{i_1}\hat{l}_{i_2}\cdots} + \Gamma^{\text{eff,1L,light}}_{\hat{l}_{i_1}\hat{l}_{i_2}\cdots} + \mathcal{O}(2\text{ loop}) \;. \quad (4.43)$$

Terms of two-loop order are neglected, since one is only interested in one-loop threshold corrections. In Eq. (4.43) all external momenta are set to zero. The same equation holds for the first derivatives of the 1PI two-point functions. Imposing all matching conditions yields the definitions for the K_{l_i} in terms of 1PI correlation functions and at the same time the desired relations between the parameters of the effective and full theory

$$\hat{\rho}_i = f_i(\rho_1, \ldots, \rho_p, K_{l_1}, \ldots, K_{l_q}) \;. \quad (4.44)$$

Per construction $\mathcal{L}^{\text{eff}}$ consists of all terms of $\mathcal{L}^{\text{full}}$ which contain only light fields, plus loop corrections (see Eq. (4.12)). This implies that Eq. (4.44) can be written as

$$\hat{\rho}_i = \rho_i + f_i^{(1\text{ loop})}(\rho_1, \ldots, \rho_p, K_{l_1}, \ldots, K_{l_q}) \;, \quad (4.45)$$

where the one-loop contribution $f_i^{(1\text{ loop})}$ in general depends on all parameter ρ_i and the renormalization constants K_{l_i}. If furthermore the effective and the full theory are renormalized in the same renormalization scheme, one has

$$\Gamma^{\text{full,1L,light}}_{l_{i_1}l_{i_2}\cdots} = \Gamma^{\text{eff,1L,light}}_{\hat{l}_{i_1}\hat{l}_{i_2}\cdots} + \mathcal{O}(2\text{ loop}) \quad (4.46)$$

so that the light loop parts on both sides of Eq. (4.43) cancel against each other.

4.3 Matching the E$_6$SSM gauge couplings to the Standard Model

4.3.1 General matching of unbroken gauge theories

In order to calculate threshold corrections for a gauge coupling, the matching procedure from Sec. 4.2 is applied to a gauge theory with a non-abelian gauge group $SU(N)$. The Lagrangian of the full theory reads

$$\mathcal{L}^{\text{full}} = \bar{\psi}_l\left(\mathrm{i}\not{D} - m\right)\psi_l + \bar{\psi}_h\left(\mathrm{i}\not{D} - M\right)\psi_h - \frac{1}{4}F_a^{\mu\nu}F^a_{\mu\nu} - \frac{1}{2\xi}(\partial^\mu A^a_\mu)^2 - \bar{u}^a\partial^\mu\bar{D}^{ab}_\mu u^b \quad (4.47)$$

where

$$D_{\mu,ij} = \partial_\mu\delta_{ij} + \mathrm{i}gA^a_\mu T^a_{ij} \quad (4.48)$$

$$\bar{D}^{ab}_\mu = \partial_\mu\delta^{ab} + gf^{abc}A^c_\mu \quad (4.49)$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gf^{abc}A^b_\mu A^c_\nu \quad (4.50)$$

and T^a are the generators of the gauge transformation (see Appendix B.2). The fields u^a and $\bar{u}^a$ are ghost fields, which absorb unphysical gauge degrees of freedom. The spinorial field ψ_h is assumed to be heavy and is integrated out. The effective Lagrangian then reads

$$\mathcal{L}^{\text{eff}} = \bar{\hat{\psi}}_l \left(\mathrm{i}\hat{\not{D}} - \hat{m}\right) \hat{\psi}_l - \frac{1}{4}\hat{F}_a^{\mu\nu}\hat{F}_{\mu\nu}^a - \frac{1}{2\hat{\xi}}(\partial^\mu \hat{A}_\mu^a)^2 - \bar{\hat{u}}^a \partial^\mu \hat{\hat{D}}_\mu^{ab}\hat{u}^b \tag{4.51}$$

where the modified covariant derivatives and field strength tensors are given by

$$\hat{D}_{\mu,ij} = \partial_\mu \delta_{ij} + \mathrm{i}\hat{g}\hat{A}_\mu^a T_{ij}^a \tag{4.52}$$

$$\hat{\hat{D}}_\mu^{ab} = \partial_\mu \delta^{ab} + \hat{g} f^{abc}\hat{A}_\mu^c \tag{4.53}$$

$$\hat{F}_{\mu\nu}^a = \partial_\mu \hat{A}_\nu^a - \partial_\nu \hat{A}_\mu^a - \hat{g} f^{abc}\hat{A}_\mu^b \hat{A}_\nu^c \ . \tag{4.54}$$

In this case it is sufficient to match the three 1PI correlation functions

$$\partial_p \, \Gamma^{\text{full}}_{\psi_l\bar{\psi}_l}(p,-p)\Big|_{p=0} = \partial_p \, \Gamma^{\text{eff}}_{\psi_l\bar{\psi}_l}(p,-p)\Big|_{p=0} \tag{4.55}$$

$$\partial_{k^2} \, \Gamma^{\text{full}}_{A_\mu^a A_\nu^b}(k,-k)\Big|_{k^2=0} = \partial_{k^2} \, \Gamma^{\text{eff}}_{A_\mu^a A_\nu^b}(k,-k)\Big|_{k^2=0} \tag{4.56}$$

$$\Gamma^{\text{full}}_{A_\mu^a \psi_l\bar{\psi}_l}(k,p,-(p+k))\Big|_{p=k=0} = \Gamma^{\text{eff}}_{A_\mu^a \psi_l\bar{\psi}_l}(k,p,-(p+k))\Big|_{p=k=0} \ . \tag{4.57}$$

In the following the full and the effective theory are renormalized in the same renormalization scheme. The required relative field renormalizations read

$$\hat{A}_\mu^a = A_\mu^a\left(1 + \frac{1}{2}K_A\right) , \qquad \hat{\psi}_l = \psi_l\left(1 + \frac{1}{2}K_{\psi_l}\right) \ . \tag{4.58}$$

At first the matching condition for the 1PI 2-point functions of the light fermion fields ψ_l is examined. Decomposing the correlation functions into tree-level and loop parts and inserting the known tree-level expressions

$$\Gamma^{\text{full,tree}}_{\psi_l\bar{\psi}_l}(p) = \not{p} - m \tag{4.59}$$

$$\Gamma^{\text{eff,tree}}_{\hat{\psi}_l\bar{\hat{\psi}}_l}(p) = \not{p} - \hat{m} \tag{4.60}$$

into Eq. (4.55) one gets a definition for K_{ψ_l}

$$K_{\psi_l} = \frac{\partial}{\partial \not{p}} \, \Gamma^{\text{full,1L,heavy}}_{\psi_l\bar{\psi}_l}\Big|_{p^\nu=0} \ . \tag{4.61}$$

Analogous one obtains from Eq. (4.56) a definition for K_A

$$\delta_{ab}K_A = -\frac{\partial}{\partial k^2} \, \Gamma^{\text{full,1L,heavy}}_{A_\mu^a A_\nu^b,\text{T}}\Big|_{k^2=0} \ , \tag{4.62}$$

where the transverse tree-level correlation functions for the massless fields A_μ^a are given by

$$\Gamma^{\text{full,tree}}_{A^a_\mu A^b_\nu,\text{T}}(k) = \Gamma^{\text{eff,tree}}_{\hat{A}^a_\mu \hat{A}^b_\nu,\text{T}}(k) = -k^2\delta_{ab} \ . \tag{4.63}$$

From the matching of the 3-point function (4.57) at $p = 0$, one obtains the relation between the gauge couplings in the full and effective theory. For this one inserts the tree-level correlation functions

$$\Gamma^{\text{full,tree}}_{A^a_\mu \psi_l \bar{\psi}_l} = -g\gamma^\mu T_a \tag{4.64}$$

$$\Gamma^{\text{eff,tree}}_{\hat{A}^a_\mu \hat{\psi}_l \bar{\hat{\psi}}_l} = -\hat{g}\gamma^\mu T_a \tag{4.65}$$

into Eq. (4.57) and writes the unknown one-loop 1PI correlation function in the form

$$\Gamma^{\text{full,1L,heavy}}_{A^a_\mu \psi_l \bar{\psi}_l}\Big|_{p=k=0} = -g\gamma^\mu T^a K_1 \ . \tag{4.66}$$

This yields the desired relation between the coupling of the effective and the full theory in terms of loop amplitudes

$$\hat{g} = g\left(1 + K_1 - K_{\psi_l} - \frac{1}{2}K_A\right) \ . \tag{4.67}$$

Slavnov–Taylor identity

In order to simplify Eq. (4.67) further one can make use of the Slavnov–Taylor identity between the renormalization constants [43]

$$\frac{1 + K_1}{1 + K_{\psi_l}} = \frac{1 + K_7}{1 + K_u} \ , \tag{4.68}$$

where K_u is the relative field renormalization of the ghost field $\hat{u}^a = (1 + \frac{1}{2}K_u)u^a$ and K_7 contains loop corrections from heavy particles to the ghost–ghost–gluon vertex

$$\mathrm{i}\Gamma^{\text{full,1L,heavy}}_{u_c A^\mu_b \bar{u}_a}(k_1, k_2, -k)\Big|_{k_1=k_2=k=0} = -g f_{bac} k_\mu K_7 \ . \tag{4.69}$$

Since one integrates out only fermions from the full theory, which do not couple to ghost fields, one obtains at the one-loop level

$$K_7 = K_u = 0 \qquad \Rightarrow \qquad K_1 = K_{\psi_l} \ . \tag{4.70}$$

This leads to the simplified relation

$$\hat{g} = g\left(1 - \frac{1}{2}K_A\right) \tag{4.71}$$

that only depends on K_A. Therefore only vector boson self-energies need to be calculated to obtain the threshold corrections.

The shown matching procedure also holds for a gauge theory with light and heavy scalars ϕ_l, ϕ_h, where the heavy one is integrated out. The resulting threshold correction then reads

$$\hat{g} = g\left(1 + \tilde{K}_1 - K_{\phi_l} - \frac{1}{2}K_A\right) \tag{4.72}$$

where $\tilde{K}_1$ and K_{ϕ_l} are related to $\Gamma_{A^a_\mu \phi_l \phi_l^*}$ and $\Gamma_{\phi_l \phi_l^*}$, respectively. The field renormalization constant K_A contains loop contributions from the heavy scalars in this case. Even the Slavnov–Taylor identity $\tilde{K}_1 = K_{\phi_l}$ is fulfilled at one-loop level, because the scalars do not couple to ghost fields. Therefore in this case the threshold correction to the gauge coupling also only depends on K_A, which again leads to Eq. (4.71).

4.3.2 Gauge group $SU(3)_\mathrm{c}$

Since the $SU(3)_\mathrm{c}$ gauge invariance is unbroken in the Standard Model as well as in the E$_6$SSM, one can use the above results to calculate the threshold correction to g_3 between both models. Equation (4.70) implies that the Standard Model coupling g_3^{SM} is related to the E$_6$SSM coupling $g_3^{\mathrm{E_6SSM}}$ by the gluon self-energy only. Therefore only colored particles contribute to the threshold correction. These are the left and right-handed squarks, the exotic quarks, their superpartners and the gluino.

Renormalization scheme and renormalization conditions

By a renormalization transformation of the theory's fields and parameters the n-point correlation function can be split into divergent loop graphs $\tilde{\Gamma}$ and counterterms $\delta\Gamma$

$$\Gamma_{F_{i_1} F_{i_2} \cdots} = \tilde{\Gamma}_{F_{i_1} F_{i_2} \cdots} + \delta\Gamma_{F_{i_1} F_{i_2} \cdots} \tag{4.73}$$

The counterterms must be chosen such that Γ is finite. The specific choice of $\delta\Gamma$ is called renormalization scheme.

In the following calculation the $\overline{\mathrm{MS}}$ renormalization scheme is used [41, 42]. In this scheme, the divergent loop graphs are evaluated in dimensional regularization, where four-dimensional integrals are evaluated in $D = 4 - 2\epsilon$ dimensions and a regulator mass μ is introduced to restore the mass dimension of the integrals. The $\overline{\mathrm{MS}}$ counterterms $\delta\Gamma^{\overline{\mathrm{MS}}}$ are then chosen in such a way, that they absorb all parts of the amplitude $\tilde{\Gamma}$, which are proportional to the divergence $\Delta = \frac{1}{\epsilon} - \gamma_\mathrm{E} + \log 4\pi$

$$\delta\Gamma^{\overline{\mathrm{MS}}}_{F_{i_1} F_{i_2} \cdots} = -\left.\tilde{\Gamma}_{F_{i_1} F_{i_2} \cdots}\right|_\Delta . \tag{4.74}$$

For the matched correlation functions (4.55)–(4.57) one has in particular

$$\delta\Gamma^{\mathrm{full,1L,heavy}}_{\psi\bar{\psi}} = -\left.\tilde{\Gamma}^{\mathrm{full,1L,heavy}}_{\psi\bar{\psi}}\right|_\Delta \tag{4.75}$$

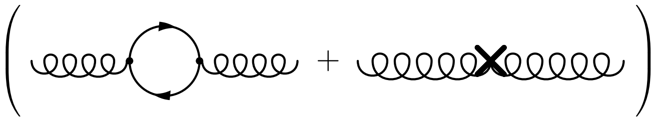
$$\delta\Gamma^{\mathrm{full,1L,heavy}}_{A^a_\mu A^b_\nu} = -\left.\tilde{\Gamma}^{\mathrm{full,1L,heavy}}_{A^a_\mu A^b_\nu}\right|_\Delta \tag{4.76}$$

$$\delta\Gamma^{\text{full,1L,heavy}}_{A^a_\mu\psi\bar{\psi}} = -\left.\tilde{\Gamma}^{\text{full,1L,heavy}}_{A^a_\mu\psi\bar{\psi}}\right|_{\Delta} . \tag{4.77}$$

Fermionic and scalar contributions to K_A

The general Dirac fermion contribution to K_A is given by the amplitude

$$\mathrm{i}\Gamma^{\text{full,1L,heavy}}_{A^a_\mu A^b_\nu,\mathrm{T}} = \mathrm{P}^{\mu\nu}_{\mathrm{T}} \tag{4.78}$$



$$= -\mathrm{i}p^2\delta_{ab}\Pi^{\text{fermion}}(p^2) , \tag{4.79}$$

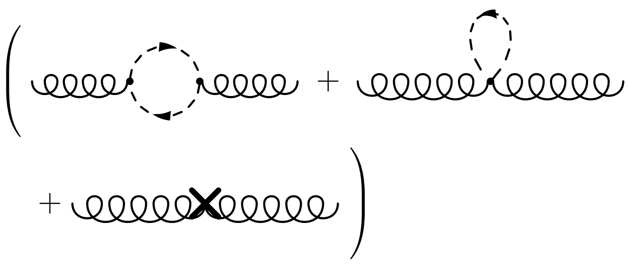
where the transversal projector $\mathrm{P}^{\mu\nu}_{\mathrm{T}}$ is defined in Appendix A.1. When evaluated at $p = 0$ in the $\overline{\text{MS}}$ renormalization scheme, Eq. (4.79) yields K_A for fermion loops

$$K_A^{\text{fermion}} = \Pi^{\text{fermion}}(p^2 = 0) = -\frac{8g_3^2}{3(4\pi)^2}C_f \log\frac{m_f}{\mu} . \tag{4.80}$$

In case of a Majorana fermion instead of a Dirac fermion within the loop, the right-hand side of Eq. (4.80) would have an additional symmetry factor of 1/2. The constant C_f is an invariant of the group representation of the fermion field and defined in Appendix B.2.

Charged scalar fields contribute to K_A via the sum of the following amplitudes

$$\mathrm{i}\Gamma^{\text{full,1L,heavy}}_{A^a_\mu A^b_\nu,\mathrm{T}} = \mathrm{P}^{\mu\nu}_{\mathrm{T}} \tag{4.81}$$



$$= -\mathrm{i}p^2\delta_{ab}\Pi^{\text{scalar}}(p^2) , \tag{4.82}$$

which yields for K_A in the $\overline{\text{MS}}$ scheme

$$K_A^{\text{scalar}} = \Pi^{\text{scalar}}(p^2 = 0) = -\frac{2g_3^2}{3(4\pi)^2}C_s \log\frac{m_s}{\mu} . \tag{4.83}$$

Here C_s is the group representation invariant for the scalar field (Appendix B.2). Note, that in case of a real instead of a charged scalar field within the loops, the right-hand side of Eq. (4.83) would have an additional symmetry factor of 1/2.

Result for the matching of the Standard Model to the E$_6$SSM

Combining the results from Eq. (4.83) and (4.80), one can write the general threshold correction for a gauge coupling in a non-abelian gauge theory with heavy fermions f

and scalars s as

$$\hat{g}^{\overline{\text{MS}}} = g^{\overline{\text{MS}}} + \frac{\left(g^{\overline{\text{MS}}}\right)^3}{(4\pi)^2}\left[\sum_f \frac{4}{3} C_f \log\frac{m_f}{\mu} + \sum_s \frac{1}{3} C_s \log\frac{m_s}{\mu}\right] . \tag{4.84}$$

This result is in agreement with a more general threshold correction which also includes heavy gauge bosons [44]

$$\begin{aligned}\hat{g}^{\overline{\text{MS}}} = g^{\overline{\text{MS}}} + \frac{\left(g^{\overline{\text{MS}}}\right)^3}{(4\pi)^2}\Bigg[&\sum_f \frac{4}{3} C_f \log\frac{m_f}{\mu} + \sum_s \frac{1}{3} C_s \log\frac{m_s}{\mu} \\ &+ \sum_a \sum_{A,C} \frac{1}{6} f^{aAC} f^{aAC}\left(1 - 21\log\frac{(m_v)_{AA}}{\mu}\right)\Bigg] .\end{aligned} \tag{4.85}$$

The indices A and C sum over the broken gauge group generators which are associated to the massive vector bosons v with masses $(m_v)_{AA}$.

Equation (4.85) yields the complete one-loop threshold corrections to g_3 for the matching of the Standard Model to the E$_6$SSM. In this case only colored particles which are not included in the Standard Model contribute. These are the gluino, the squarks and the exotic particles. The threshold correction to g_3 then reads

$$\begin{aligned}g_3^{\overline{\text{MS}},\text{SM}} = g_3^{\overline{\text{MS}},\text{E}_6\text{SSM}} + \frac{\left(g_3^{\overline{\text{MS}},\text{E}_6\text{SSM}}\right)^3}{(4\pi)^2}\Bigg[&2\log\frac{m_{\tilde{g}}}{\mu} + \frac{1}{6}\sum_{\tilde{q}\in\{\tilde{u},\tilde{d}\}}\sum_{i=1}^{3}\sum_{k=1}^{2}\log\frac{m_{\tilde{q}_{ik}}}{\mu} \\ &+ \frac{2}{3}\sum_{i=1}^{3}\log\frac{m_{x_i}}{\mu} + \frac{1}{6}\sum_{i=1}^{3}\sum_{k=1}^{2}\log\frac{m_{\tilde{x}_{ik}}}{\mu}\Bigg] .\end{aligned} \tag{4.86}$$

The index i sums over all generations and k sums over the mass eigenstates. It was used that the squarks and the exotic particles are in the fundamental and the gluino is in the adjoint representation of $SU(3)_\text{c}$. Thus their group representation invariants are

$$C_{\tilde{g}} = 3\,, \qquad C_{\tilde{q}_{ik}} = C_{\tilde{x}_{ik}} = C_{x_i} = \frac{1}{2}\,. \tag{4.87}$$

Since the gluino is a Majorana fermion, a symmetry factor 1/2 was added. Note that the squark masses in Eq. (4.86) are diagonalized mass eigenvalues. Mixing matrices do not appear in the result, because they drop out explicitly in the calculation of the Feynman rules (see Appendix B.10).

Relation to the beta function

The beta function describes the dependency of the gauge coupling on the renormalization scale μ. It is defined by the differential equation

$$\frac{\mathrm{d}g}{\mathrm{d}t} = \frac{g^3}{(4\pi)^2}\beta \,. \tag{4.88}$$

The relation of the threshold correction to the beta function can be obtained from the general result Eq. (4.85) by differentiating with respect to $t = \log \mu$

$$\frac{\mathrm{d}g}{\mathrm{d}t} - \frac{\mathrm{d}\hat{g}}{\mathrm{d}t} = \frac{g^3}{(4\pi)^2}\left[\sum_f \frac{4}{3}C_f + \sum_s \frac{1}{3}C_s - \sum_a \sum_{A,C} \frac{21}{6} f^{aAC} f^{aAC}\right] . \tag{4.89}$$

Using the definition of the beta function (4.88) yields

$$\beta^{\text{full}} - \beta^{\text{eff}} = \sum_f \frac{4}{3}C_f + \sum_s \frac{1}{3}C_s - \sum_a \sum_{A,C} \frac{21}{6} f^{aAC} f^{aAC} \,. \tag{4.90}$$

The threshold corrections on the right-hand side of Eq. (4.90) are exactly the contributions from the heavy particles to β^{eff}, which are missing in the effective theory. The left-hand side is the change of the slope of $g(\mu)$ when switching from the full to the effective theory. The dependency of this change on μ is then given by the μ dependency of threshold corrections on the right-hand side.

4.3.3 Gauge group $SU(2)_{\mathrm{L}} \times U(1)_Y$

The calculation of threshold corrections for the couplings g_1 and g_2 between the Standard Model and the E$_6$SSM is more involved, because the gauge group $SU(2)_{\mathrm{L}} \times U(1)_Y$ is spontaneously broken to $U(1)_{\mathrm{em}}$. From the relations

$$g_1 = \frac{e}{c_W}, \qquad g_2 = \frac{e}{s_W}, \qquad c_W = \frac{m_W}{m_Z} \tag{4.91}$$

(see Section 2.3 and 3.2.5) one can see that the threshold corrections for g_1 and g_2 are related to those of the $W^{\pm}$ and Z boson masses and the gauge coupling e of the remaining gauge symmetry. Therefore the following matching conditions are imposed in order to obtain threshold corrections for e, m_Z and m_W

$$\partial^n_{k^2}\, \Gamma^{\text{full}}_{W^+_\mu W^-_\nu}(k,-k)\Big|_{k^2=0} = \partial^n_{k^2}\, \Gamma^{\text{eff}}_{W^+_\mu W^-_\nu}(k,-k)\Big|_{k^2=0} \qquad (n=0,1) \tag{4.92}$$

$$\partial^n_{k^2}\, \Gamma^{\text{full}}_{Z_\mu Z_\nu}(k,-k)\Big|_{k^2=0} = \partial^n_{k^2}\, \Gamma^{\text{eff}}_{Z_\mu Z_\nu}(k,-k)\Big|_{k^2=0} \qquad (n=0,1) \tag{4.93}$$

$$\partial_{k^2}\, \Gamma^{\text{full}}_{A_\mu A_\nu}(k,-k)\Big|_{k^2=0} = \partial_{k^2}\, \Gamma^{\text{eff}}_{A_\mu A_\nu}(k,-k)\Big|_{k^2=0} \tag{4.94}$$

$$\Gamma^{\text{full}}_{Z_\mu A_\nu}(k,-k)\Big|_{k^2=0} = \Gamma^{\text{eff}}_{Z_\mu A_\nu}(k,-k)\Big|_{k^2=0} \tag{4.95}$$

$$\partial_{p}\, \Gamma^{\text{full}}_{\psi_{Li}\bar{\psi}_{Lj}}(p,-p)\Big|_{p=0} = \partial_{p}\, \Gamma^{\text{eff}}_{\psi_{Li}\bar{\psi}_{Lj}}(p,-p)\Big|_{p=0} \tag{4.96}$$

$$\partial_p \, \Gamma^{\text{full}}_{\psi_{Ri}\bar{\psi}_{Rj}}(p,-p)\Big|_{p=0} = \partial_p \, \Gamma^{\text{eff}}_{\psi_{Ri}\bar{\psi}_{Rj}}(p,-p)\Big|_{p=0} \tag{4.97}$$

$$\Gamma^{\text{full}}_{A_\mu\psi_i\bar{\psi}_j}(k,p,-(p+k))\Big|_{k=p=0} = \Gamma^{\text{eff}}_{A_\mu\psi_i\bar{\psi}_j}(k,p,-(p+k))\Big|_{k=p=0} \; . \tag{4.98}$$

The additional matching condition for $\Gamma_{Z_\mu A_\nu}$ is necessary, because the gauge fields B_μ, $\vec{W}_\mu$ of $SU(2)_\text{L} \times U(1)_Y$ mix to A_μ, Z_μ and $W^\pm_\mu$. Introducing the following relative field renormalizations

$$\hat{W}^\pm_\mu = \left(1 + \frac{1}{2} K_{WW}\right) W^\pm_\mu \tag{4.99}$$

$$\hat{Z}_\mu = \left(1 + \frac{1}{2} K_{ZZ}\right) Z_\mu + \frac{1}{2} K_{ZA} A_\mu \tag{4.100}$$

$$\hat{A}_\mu = \left(1 + \frac{1}{2} K_{AA}\right) A_\mu + \frac{1}{2} K_{AZ} Z_\mu \tag{4.101}$$

$$\hat{\psi}_{iL} = \left(\delta_{ij} + \frac{1}{2} K^L_{\psi ij}\right) \psi_{jL} \; , \qquad \hat{\psi}_{iR} = \left(\delta_{ij} + \frac{1}{2} K^R_{\psi ij}\right) \psi_{jR} \tag{4.102}$$

and inserting them into the matching conditions (4.92)–(4.97) leads to the definition of the renormalization constants

$$K_{WW} = -\frac{\partial}{\partial k^2} \, \Gamma^{\text{full,1L,heavy}}_{W^+_\mu W^-_\nu,\text{T}}\Big|_{k^2=0} \tag{4.103}$$

$$K_{ZZ} = -\frac{\partial}{\partial k^2} \, \Gamma^{\text{full,1L,heavy}}_{Z_\mu Z_\nu,\text{T}}\Big|_{k^2=0} \tag{4.104}$$

$$K_{AA} = -\frac{\partial}{\partial k^2} \, \Gamma^{\text{full,1L,heavy}}_{A_\mu A_\nu,\text{T}}\Big|_{k^2=0} \tag{4.105}$$

$$K_{ZA} = \frac{2}{m_Z^2} \, \Gamma^{\text{full,1L,heavy}}_{A_\mu Z_\nu,\text{T}}\Big|_{k^2=0} \tag{4.106}$$

$$K^L_{\psi ij} = \frac{\partial}{\partial \not{p}} \, \Gamma^{\text{full,1L,heavy}}_{\psi_{iL}\bar{\psi}_{jL}}\Big|_{p=0} \tag{4.107}$$

$$K^R_{\psi ij} = \frac{\partial}{\partial \not{p}} \, \Gamma^{\text{full,1L,heavy}}_{\psi_{iR}\bar{\psi}_{jR}}\Big|_{p=0} \; . \tag{4.108}$$

Furthermore, Eq. (4.92) and (4.93) yield for $n = 0$ the threshold corrections for the W and Z boson masses

$$\hat{m}_V^2 = m_V^2 + \Gamma^{\text{full,1L,heavy}}_{V_\mu V_\nu,\text{T}}\Big|_{k^2=0} - m_V^2 K_{VV} \; , \qquad V \in \{W, Z\} \; . \tag{4.109}$$

The next step is to decompose the renormalized three-point function in the limit of vanishing external momenta into

$$\Gamma^{\text{full,1L,heavy}}_{A_\mu\psi_i\bar{\psi}_j}\Big|_{p=k=0} = -e\gamma^\mu Q \delta_{ij} K_1 - e\gamma^\mu Q K'_{1,ij} + e\gamma^\mu \delta_{ij} K''_1 P_L \; . \tag{4.110}$$

The first two terms on the right hand side of Eq. (4.110) are left-right diagonal and the first and last term are diagonal in the fermion flavors. Inserting the tree level correlation functions

$$\Gamma^{\text{full,tree}}_{A_\mu \psi_i \bar{\psi}_j} = -e\gamma^\mu Q \delta_{ij} \tag{4.111}$$

$$\Gamma^{\text{eff,tree}}_{\hat{A}_\mu \hat{\psi}_i \hat{\bar{\psi}}_j} = -\hat{e}\gamma^\mu Q \delta_{ij} \tag{4.112}$$

$$\Gamma^{\text{eff,tree}}_{\hat{Z}_\mu \hat{\psi}_i \hat{\bar{\psi}}_j} = -\hat{e}\gamma^\mu Q \frac{s_W}{c_W} \delta_{ij} + \hat{e}\gamma^\mu \frac{I_W^3}{s_W c_W} \delta_{ij} P_L \tag{4.113}$$

as well as the decomposed three-point function (4.110) into the matching condition (4.98), one arrives at the general threshold correction for the electromagnetic coupling e

$$\begin{aligned} \hat{e}Q = eQ &\left(1 + K_1 - K_\psi^L P_L - K_\psi^R P_R - \frac{1}{2} K_{AA} - \frac{s_W}{2c_W} K_{ZA}\right) \\ &+ e\left(P_L \frac{I_W^3}{2s_W c_W} K_{ZA} - P_L K_1''\right) . \end{aligned} \tag{4.114}$$

Here $I_W^3 = \tau_3/2$ denotes the third component of the weak isospin of $\hat{\psi}_i$ and it was set $K_\psi^{L,R} \equiv K_{\psi ii}^{L,R}$.

Ward–Takahashi identity in the Standard Model

It is assumed here, that the Z' does not mix with the other gauge bosons. In this case one can simplify Eq. (4.114) further by using the Standard Model analogue to the QED Ward–Takahashi identity [45]

$$\begin{aligned} \bar{u}(p) \frac{1}{e} \Gamma^{\text{full,1L,heavy}}_{A_\mu \psi_i \bar{\psi}_i} u(p) = &- Q \bar{u}(p) \left[\partial_{p^\mu} \Gamma^{\text{full,1L,heavy}}_{\psi_i \bar{\psi}_i}(p)\right] u(p) \\ &+ a \bar{u}(p) \gamma_\mu P_L u(p) \frac{2}{m_Z^2} \Gamma^{\text{full,1L,heavy}}_{A_\mu Z_\nu, \text{T}}(0) . \end{aligned} \tag{4.115}$$

The constant $a = I_W^3/2s_W c_W$ is the axial coupling of the Z boson to the fermion fields. In the limit of vanishing external momenta one has from Eq. (4.106)–(4.108) and (4.110)

$$\frac{2}{m_Z^2} \Gamma^{\text{full,1L,heavy}}_{A_\mu Z_\nu, \text{T}}(0) = K_{ZA} \tag{4.116}$$

$$\partial_{p^\mu} \Gamma^{\text{full,1L,heavy}}_{\psi_i \bar{\psi}_i}\Big|_{p=0} = \gamma_\mu \left(K_\psi^L P_L + K_\psi^R P_R\right) \tag{4.117}$$

$$\frac{1}{e} \Gamma^{\text{full,1L,heavy}}_{A \psi_i \bar{\psi}_i}(0,0,0) = -Q\gamma_\mu K_1 + \gamma_\mu P_L K_1'' . \tag{4.118}$$

This yields a relation between the renormalization constants

$$-QK_1 + P_L K_1'' = -Q(K_\psi^L P_L + K_\psi^R P_R) + \frac{I_W^3}{2s_W c_W} P_L K_{ZA} , \tag{4.119}$$

which implies a simplified equation for the threshold correction of the electromagnetic coupling (4.114)

$$\hat{e} = e\left(1 - \frac{1}{2}K_{AA} - \frac{s_W}{2c_W}K_{ZA}\right) . \tag{4.120}$$

This relation contains vector boson self-energies only. Note, that Eq. (4.120) is analogous to (4.67) and (4.70), but here one gets extra contributions from the A–Z mixing.

Threshold corrections for g_1 and g_2

From Eq. (4.109) and (4.120) one can now derive the threshold corrections for g_1 and g_2. For convenience the shorthand notation

$$\hat{m}_V^2 = m_V^2 + \Delta m_V^2 , \qquad V \in \{W, Z\} \tag{4.121}$$

$$\hat{e} = e + \Delta e \tag{4.122}$$

of (4.109) and (4.120) is introduced, where

$$\Delta m_V^2 = \Gamma^{\text{full,1L,heavy}}_{V_\mu V_\nu,\text{T}}\Big|_{k^2=0} + m_V^2 \frac{\partial}{\partial k^2}\Gamma^{\text{full,1L,heavy}}_{V_\mu V_\nu,\text{T}}\Bigg|_{k^2=0} \tag{4.123}$$

$$\Delta e = e\left(-\frac{1}{2}K_{AA} - \frac{s_W}{2c_W}K_{ZA}\right) . \tag{4.124}$$

Then one can write the threshold corrections for g_1 and g_2 in the form

$$\hat{g}_1 = \frac{\hat{e}}{\hat{c}_W} = \frac{\hat{e}\,\hat{m}_Z}{\hat{m}_W} = g_1\left(1 + \frac{\Delta e}{e} + \frac{1}{2}\frac{\Delta m_Z^2}{m_Z^2} - \frac{1}{2}\frac{\Delta m_W^2}{m_W^2}\right) \tag{4.125}$$

$$\hat{g}_2 = \frac{\hat{e}}{\hat{s}_W} = \frac{\hat{e}}{\sqrt{1 - \frac{\hat{m}_W^2}{\hat{m}_Z^2}}} = g_2\left(1 + \frac{\Delta e}{e} - \frac{c_W^2}{s_W^2}\frac{\Delta m_Z^2}{2m_Z^2} + \frac{c_W^2}{s_W^2}\frac{\Delta m_W^2}{2m_W^2}\right) . \tag{4.126}$$

Renormalization conditions

For convenience the $\overline{\text{MS}}$ renormalization scheme is used again (see Sec. 4.3.2) and the counterterms are set such that they absorb only terms which are proportional to Δ

$$\delta\Gamma^{\text{full,1L,heavy}}_{F_1F_2} = -\,\tilde{\Gamma}^{\text{full,1L,heavy}}_{F_1F_2}\Big|_{\Delta} , \qquad F_i \in \{A_\mu, Z_\mu, W_\mu^\pm\} . \tag{4.127}$$

Resulting threshold corrections for g_1 and g_2 when matching the Standard Model to the E$_6$SSM

The above procedure is now applied to the matching of the Standard Model to the E$_6$SSM. For simplicity only particles are included in the calculation which are expected to have dominant loop contributions to electroweak observables. These are the top and the bottom quarks in the Standard Model and stop and the sbottom squarks in the MSSM [46]. The reason is that these particles break the Custodial symmetry at most due to their large mass difference. In the E_6SSM it is expected that the exotic particles x_i and $\tilde{x}_{ik}$ give the largest contributions. When integrating out these, the renormalization constants become

$$K_{AA} = -\frac{2e^2}{3(4\pi)^2} N_c \left(4 \sum_{i=1}^{3} Q_{x_i}^2 \log \frac{m_{x_i}}{\mu} + \sum_{i=1}^{3} \sum_{k=1}^{2} Q_{\tilde{x}_{ik}}^2 \log \frac{m_{\tilde{x}_{ik}}}{\mu} \right) \tag{4.128}$$

$$K_{ZZ} = \frac{s_W^2}{c_W^2} K_{AA} \tag{4.129}$$

$$K_{WW} = 0 \tag{4.130}$$

$$K_{ZA} = 0 \; . \tag{4.131}$$

In this calculation the Feynman rules given in Appendix B.10 as well as the results for fermionic and scalar loops (4.80), (4.83) were used. The threshold corrections for g_1 and g_2 then read

$$\begin{aligned} g_1^{\overline{\text{MS}},\text{SM}} &= g_1^{\overline{\text{MS}},\text{E}_6\text{SSM}} + \frac{\left(g_1^{\overline{\text{MS}},\text{E}_6\text{SSM}}\right)^3}{(4\pi)^2} \left[\frac{4}{3} N_c \sum_{i=1}^{3} \left(\frac{Y_{x_i}}{2}\right)^2 \log \frac{m_{x_i}}{\mu} \right. \\ &\qquad \left. + \frac{1}{3} N_c \sum_{i=1}^{3} \sum_{k=1}^{2} \left(\frac{Y_{\tilde{x}_{ik}}}{2}\right)^2 \log \frac{m_{\tilde{x}_{ik}}}{\mu} \right] \end{aligned} \tag{4.132}$$

$$g_2^{\overline{\text{MS}},\text{SM}} = g_2^{\overline{\text{MS}},\text{E}_6\text{SSM}} \; , \tag{4.133}$$

where $N_c = 3$ is the color factor and the Hypercharges are listed in Table 3.1. The Equations (4.132) and (4.133) are in agreement with the general result (4.85), whereas the group factors in the fundamental representations of $U(1)_Y$ and $SU(2)_\text{L}$ are

$$U(1)_Y \qquad \begin{cases} C_{x_i} = \left(\frac{Y_{x_i}}{2}\right)^2 \\ C_{\tilde{x}_{ik}} = \left(\frac{Y_{\tilde{x}_{ik}}}{2}\right)^2 \end{cases} \tag{4.134}$$

$$SU(2)_\text{L} \qquad \begin{cases} C_{x_i} = 0 \\ C_{\tilde{x}_{ik}} = 0 \; . \end{cases} \tag{4.135}$$

5 Implementation and results

This chapter describes the procedure suggested in [1] to calculate $\mathrm{cE_6SSM}$ particle spectra. It is shown that the inclusion of threshold corrections into this procedure is necessary in order to increase the precision of the particle spectrum prediction. The in Chapter 4 calculated threshold corrections are implemented into the particle spectrum generator and are numerically evaluated. Finally more precise $\mathrm{cE_6SSM}$ particle spectra are calculated for a set of interesting parameter points.

5.1 The particle spectrum generator

5.1.1 Effective field theory approach

In order to calculate the parameters and masses of the $\mathrm{cE_6SSM}$, one has to fulfill high scale and low scale constraints. At the unification scale M_X the model is defined by the set of Yukawa couplings $\lambda_i(M_X)$, $\kappa_i(M_X)$ and the universal soft parameters m_0^2, $M_{1/2}$, A_0. At the low scale the $\mathrm{cE_6SSM}$ parameters must be consistent with measured low-energy observables, e. g., m_Z, m_W, G_μ etc.

This can be solved by iterating between the high and the low scale and using the effective field theory approach to match to the low-energy observables, see Figure 5.1. One starts at the unification scale M_X and inputs all high scale parameters into the model. Then one uses the $\mathrm{E_6SSM}$ renormalization group running to evolve all model parameters down to an intermediate matching scale $T_{\mathrm{E_6SSM}}$. At this scale, one integrates out all $\mathrm{E_6SSM}$ particles, which are not part of the Standard Model and uses the threshold corrections to calculate the Standard Model parameters from the given $\mathrm{E_6SSM}$ parameters. After this matching is done, one can use the Standard Model renormalization group equations to evolve the parameters to a suitable low-energy scale and match to measured low-energy values.

5.1.2 Numerical procedure

In a recent publication [1] a procedure for calculating $\mathrm{cE_6SSM}$ particle spectra was presented and first spectra were shown. For the calculation a particle spectrum generator was used, which is based on `SOFTSUSY` 2.0.5 [47]. This generator uses the above discussed effective field theory approach to calculate particle masses. As input the program gets a $\mathrm{cE_6SSM}$ parameter point specified by

$$\lambda_i(M_X), \kappa_i(M_X), s, \tan\beta\ . \tag{5.1}$$

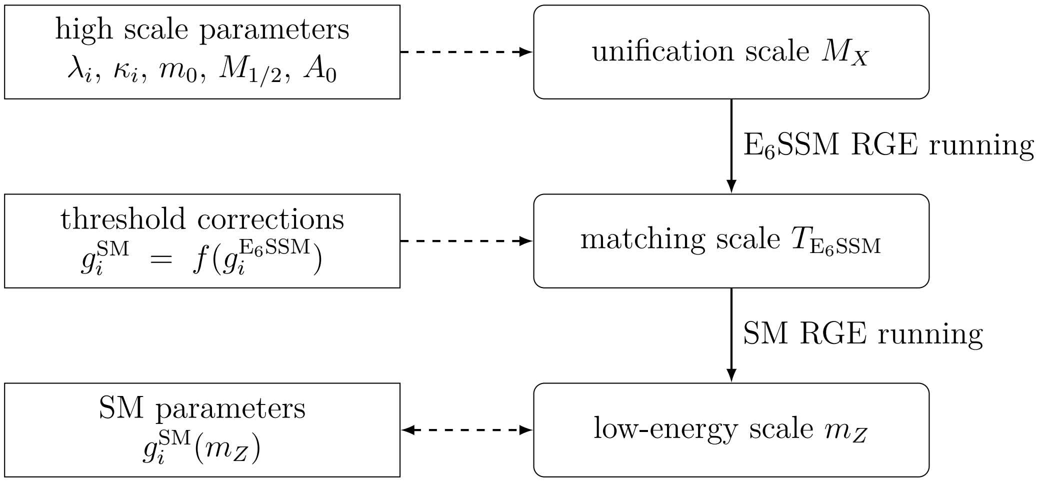


Figure 5.1: Effective field theory approach to calculate cE$_6$SSM model parameters, which are consistent with low-energy Standard Model observables.

The procedure of calculating the mass spectrum for this parameter point consists of four parts. At first the gauge and Yukawa couplings in the cE$_6$SSM are iteratively calculated such that they are consistent with the program input parameters (5.1), experimental measured quantities at low energies and gauge coupling unification. In particular this works as follows.

1. Set the Standard Model $\overline{\text{MS}}$ gauge and Yukawa couplings at renormalization scale $\mu = m_Z$ to measured experimental values [3].

2. Evolve the gauge and Yukawa couplings from m_Z to the intermediate matching scale $T_{\text{E}_6\text{SSM}}$ using the two-loop Standard Model renormalization group equations (RGE). The scale $T_{\text{E}_6\text{SSM}}$ is chosen to be of the order of the heavy E$_6$SSM particles, which are integrated out. This ensures that loop corrections are small and the perturbation expansion is valid.

3. a) Convert the Standard Model Yukawa couplings to E$_6$SSM Yukawa couplings using

$$f^e_{\text{E}_6\text{SSM}} = \frac{f^e_{\text{SM}}}{\cos\beta} , \qquad f^d_{\text{E}_6\text{SSM}} = \frac{f^d_{\text{SM}}}{\cos\beta} , \qquad f^u_{\text{E}_6\text{SSM}} = \frac{f^u_{\text{SM}}}{\sin\beta} \tag{5.2}$$

   b) Convert the Standard Model gauge couplings to E$_6$SSM gauge couplings via

$$g_i^{\overline{\text{DR}},\text{E}_6\text{SSM}} = g_i^{\overline{\text{MS}},\text{SM}} , \qquad (i = 1, 2, 3) \tag{5.3}$$

   The definition of the $\overline{\text{DR}}$ renormalization scheme is explained in Section 5.1.4.

   c) Estimate low energy values for the Yukawa couplings $\lambda_i(T_{\text{E}_6\text{SSM}})$, $\kappa_i(T_{\text{E}_6\text{SSM}})$.

The calculated E_6SSM gauge and Yukawa couplings g_i, f^k form a set of low-energy boundary conditions at scale $T_{\text{E}_6\text{SSM}}$.

4. Evolve all E_6SSM parameters, except the soft parameters, from $T_{\text{E}_6\text{SSM}}$ to the unification scale M_X using two-loop E_6SSM RGEs.

5. a) Set Yukawa couplings $\lambda_i(M_X)$, $\kappa_i(M_X)$ to program input values.

   b) Set $g_N = g_0/\sqrt{40}$, where g_0 is defined by $\sqrt{\frac{5}{3}}\, g_1(M_X)$.

6. Perform an iteration between M_X and $T_{\text{E}_6\text{SSM}}$ to obtain values for the gauge and Yukawa couplings, which are consistent with gauge coupling unification, low energy boundary conditions $f^k(T_{\text{E}_6\text{SSM}})$, $g_i(T_{\text{E}_6\text{SSM}})$ and high energy boundary conditions $\lambda_i(M_X)$, $\kappa_i(M_X)$.

Afterwards the dependency of the low energy soft mass parameters on the GUT scale values m_0^2, $M_{1/2}$, A_0 is calculated. For that purpose the low energy soft parameters are expressed in terms of m_0^2, $M_{1/2}$, A_0

$$m_i^2(t) = a_i(t)m_0^2 + b_i(t)M_{1/2}^2 + c_i(t)A_0M_{1/2} + d_i(t)A_0^2 \tag{5.4}$$

$$A_i(t) = e_i(t)A_0 + f_i(t)M_{1/2} \tag{5.5}$$

$$M_i(t) = p_i(t)A_0 + q_i(t)M_{1/2} \tag{5.6}$$

and the coefficients $a_i(t)$, …, $q_i(t)$ are calculated numerically at $t = \log T_{\text{E}_6\text{SSM}}/M_X$. The obtained values for the low energy soft parameters $m_i^2(t)$, $M_i(t)$, $A_i(t)$ are then combined with the EWSB conditions

$$\frac{\partial V}{\partial v_1} = \frac{\partial V}{\partial v_2} = \frac{\partial V}{\partial s} = 0 \tag{5.7}$$

to determine valid sets of values for the universal soft parameters $\{m_0^2, M_{1/2}, A_0\}$ which are consistent with EWSB. It is important to note that in general EWSB is not guaranteed in the cE_6SSM, i. e., solutions for $\{m_0^2, M_{1/2}, A_0\}$ from Eq. (5.7) are not always found. But for sufficiently large values of κ_i, the soft parameter $m_{s_3}^2$ always gets negative at low energies, which triggers the EWSB [1]. Finally the masses of all particles are calculated for each set of $\{m_0^2, M_{1/2}, A_0\}$.

5.1.3 Dependence on the matching scale

It is important to note, that the choice of the matching scale $T_{\text{E}_6\text{SSM}}$ is absolutely arbitrary. Physical quantities can not depend on this choice, because $T_{\text{E}_6\text{SSM}}$ is an unphysical renormalization scale. However, since one stops the perturbation expansion at a finite order n, a matching scale dependence can occur which is of the order $(n + 1)$.

The particle spectrum generator used in [1] uses two-loop renormalization group running for the gauge and Yukawa couplings, but it misses proper threshold corrections for all of them. Instead, it does a trivial conversion, for instance

$$g_i^{\overline{\text{DR}},\text{E}_6\text{SSM}} = g_i^{\overline{\text{MS}},\text{SM}} , \qquad (i = 1, 2, 3) . \tag{5.8}$$

Because of these missing threshold corrections, a one and two-loop dependence of physical quantities on $T_{\text{E}_6\text{SSM}}$ occurs. As was stated in Section 4.3.2 this effect comes from neglecting the influence of the heavy particles on the parameters of the effective theory at one and two-loop level. If the full one-loop threshold corrections would be included, only a two-loop dependence of the calculated physical quantities on the matching scale would remain. This effect is studied in the following sections by including proper threshold corrections in the above described procedure.

5.1.4 The implemented threshold corrections

In order to increase the precision of the predicted masses, the following one-loop threshold corrections are implemented in Eq. (5.3) of the particle spectrum generation procedure

$$\begin{aligned} g_1^{\overline{\text{DR}},\text{E}_6\text{SSM}} = g_1^{\overline{\text{MS}},\text{SM}} + \frac{\left(g_1^{\overline{\text{MS}},\text{SM}}\right)^3}{(4\pi)^2} \Bigg[& -\frac{4}{3} N_c \sum_{i=1}^{3} \left(\frac{Y_{x_i}}{2}\right)^2 \log \frac{m_{x_i}}{T_{\text{E}_6\text{SSM}}} \\ & -\frac{1}{3} N_c \sum_{i=1}^{3} \sum_{k=1}^{2} \left(\frac{Y_{\tilde{x}_{ik}}}{2}\right)^2 \log \frac{m_{\tilde{x}_{ik}}}{T_{\text{E}_6\text{SSM}}} \\ & -\frac{1}{3} N_c \sum_{i=1}^{3} \sum_{k=L,R} \left\{ \left(\frac{Y_{\tilde{u}_{ik}}}{2}\right)^2 \log \frac{m_{\tilde{u}_{ik}}}{T_{\text{E}_6\text{SSM}}} + \left(\frac{Y_{\tilde{d}_{ik}}}{2}\right)^2 \log \frac{m_{\tilde{d}_{ik}}}{T_{\text{E}_6\text{SSM}}} \right\} \\ & -\frac{1}{3} \sum_{i=1}^{3} \sum_{k=L,R} \left(\frac{Y_{\tilde{e}_{ik}}}{2}\right)^2 \log \frac{m_{\tilde{e}_{ik}}}{T_{\text{E}_6\text{SSM}}} - \frac{1}{3} \sum_{i=1}^{3} \left(\frac{Y_{\tilde{\nu}_{iL}}}{2}\right)^2 \log \frac{m_{\tilde{\nu}_{iL}}}{T_{\text{E}_6\text{SSM}}} \\ & -\frac{1}{3} \sum_{i=1}^{2} \sum_{p=1}^{2} \sum_{j=1}^{2} \left(\frac{Y_{h_{pi}^j}}{2}\right)^2 \log \frac{m_{h_{pi}^j}}{T_{\text{E}_6\text{SSM}}} \\ & -\frac{2}{3} \sum_{i=1}^{2} \sum_{p=1}^{2} \sum_{j=1}^{2} \left(\frac{Y_{\tilde{h}_{piL}^j}}{2}\right)^2 \log \frac{m_{\tilde{h}_{piL}^j}}{T_{\text{E}_6\text{SSM}}} \Bigg] \end{aligned} \tag{5.9}$$

$$\begin{aligned} g_2^{\overline{\text{DR}},\text{E}_6\text{SSM}} = g_2^{\overline{\text{MS}},\text{SM}} + \frac{\left(g_2^{\overline{\text{MS}},\text{SM}}\right)^3}{(4\pi)^2} \Bigg[& \frac{1}{3} - \frac{1}{6} N_c \sum_{i=1}^{3} \log \frac{m_{\tilde{q}_{iL}}}{T_{\text{E}_6\text{SSM}}} - \frac{1}{6} \sum_{i=1}^{3} \log \frac{m_{\tilde{\ell}_{iL}}}{T_{\text{E}_6\text{SSM}}} \\ & - \frac{1}{6} \sum_{i=1}^{2} \sum_{p=1}^{2} \log \frac{m_{h_{pi}}}{T_{\text{E}_6\text{SSM}}} - \frac{1}{3} \sum_{i=1}^{2} \sum_{p=1}^{2} \log \frac{m_{\tilde{h}_{piL}}}{T_{\text{E}_6\text{SSM}}} \Bigg] \end{aligned} \tag{5.10}$$

$$g_3^{\overline{\text{DR}},\text{E}_6\text{SSM}} = g_3^{\overline{\text{MS}},\text{SM}} + \frac{\left(g_3^{\overline{\text{MS}},\text{SM}}\right)^3}{(4\pi)^2} \Bigg[\frac{1}{2} - 2\log\frac{m_{\tilde{g}}}{T_{\text{E}_6\text{SSM}}} - \frac{1}{6} \sum_{\tilde{q}\in\{\tilde{u},\tilde{d}\}} \sum_{i=1}^{3}\sum_{k=1}^{2} \log\frac{m_{\tilde{q}_{ik}}}{T_{\text{E}_6\text{SSM}}} \\ - \frac{2}{3}\sum_{i=1}^{3}\log\frac{m_{x_i}}{T_{\text{E}_6\text{SSM}}} - \frac{1}{6}\sum_{i=1}^{3}\sum_{k=1}^{2}\log\frac{m_{\tilde{x}_{ik}}}{T_{\text{E}_6\text{SSM}}} \Bigg] \tag{5.11}$$

The Equations (5.9)–(5.11) contain the results from Sec. 4.3.2 and Sec. 4.3.3. In Eq. (5.9) and (5.10) approximate corrections from squarks, sleptons and the first two generation Higgses and Higgsinos are added. For all these approximate corrections mixing is neglected and the squark and slepton masses only contain the contributions from the soft supersymmetry breaking Lagrangian and the D terms. The F term contributions to the sparticle masses are small compared to the soft and D terms and are therefore neglected.

Conversion to $\overline{\textbf{DR}}$ renormalization scheme

It is known that dimensional regularization breaks supersymmetry, because the number of degrees of freedom of the gauginos and the gauge bosons does not match when treating the latter as D-dimensional objects [48]. Therefore it is not convenient to renormalize the E_6SSM in the $\overline{\text{MS}}$ renormalization scheme. Instead, the supersymmetry preserving $\overline{\text{DR}}$ scheme is used where divergent loop integrals are regularized in dimensional reduction (DRED) [49]. Here only momenta are continued from 4 to $D = 4 - 2\epsilon$ dimensions. Gauge fields and gamma matrices remain 4-dimensional objects. In this way DRED avoids the direct breaking of supersymmetry. The divergencies are then absorbed via minimal subtraction.

In the threshold corrections (5.9)–(5.11) constant terms are added in order to convert the gauge couplings from the $\overline{\text{MS}}$ to the $\overline{\text{DR}}$ renormalization scheme. This is necessary because the E_6SSM is renormalized in the $\overline{\text{DR}}$ scheme, while the Standard Model is renormalized in the $\overline{\text{MS}}$ scheme. These correction terms have the general form [50]

$$g_i^{\overline{\text{DR}}} = g_i^{\overline{\text{MS}}} + \frac{C_v}{6}\frac{\left(g_i^{\overline{\text{MS}}}\right)^3}{(4\pi)^2}\,, \qquad (i = 1,2,3) \tag{5.12}$$

where C_v is the group factor for the gauge bosons in the adjoint representation of the gauge group (see Appendix B.2). In particular one has

$$C_v = \begin{cases} 3 & \text{for } SU(3)_\text{c} \\ 2 & \text{for } SU(2)_\text{L} \\ 0 & \text{for } U(1)_Y\,. \end{cases} \tag{5.13}$$

5.1.5 Selection of parameter points

In order to study the threshold effects in the cE_6SSM one has to choose a set of input parameters $\{\lambda_i(M_X), \kappa_i(M_X), s, \tan\beta\}$. The choice must be such that the resulting particle spectrum is consistent with current exclusion bounds for the new E_6SSM particles from experiment [2, 51, 52]. In particular it is required that

$$m_{h_1} \geq 114\,\text{GeV} \tag{5.14}$$

$$m_{\tilde{l}_{1,2}} \geq 100\,\text{GeV} \qquad \tilde{l} \in \{\tilde{e}, \tilde{\mu}, \tilde{\tau}, \tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau\} \tag{5.15}$$

$$m_{\chi^\pm_{1,2}} \geq 100\,\text{GeV} \tag{5.16}$$

$$m_{\tilde{q}_{1,2}} \geq 300\,\text{GeV} \qquad \tilde{q} \in \{\tilde{u}, \tilde{d}, \tilde{s}, \tilde{c}, \tilde{b}, \tilde{t}\} \tag{5.17}$$

$$m_{Z'} \geq 860\,\text{GeV} \tag{5.18}$$

$$m_{\tilde{x}_{ik}}, m_{x_i} \geq 300\,\text{GeV} \qquad (i = 1, 2, 3), (k = 1, 2) \tag{5.19}$$

$$m_{h^{0,\pm}_{ik}}, m_{\tilde{h}^{0,\pm}_{ik}} \geq 100\,\text{GeV} \qquad (i, k = 1, 2)\;. \tag{5.20}$$

Furthermore, one looks for parameter points, where the lightest supersymmetric particle (LSP) is a neutralino, which is then a candidate for dark matter. The chosen parameter points are listed in Table 5.1.

Table 5.1: cE_6SSM parameter points

	PP1	PP2	PP3
$\tan\beta$	10	10	3
$\lambda_3(M_X)$	1.6	−2.0	−0.434
$\lambda_{1,2}(M_X)$	2.6	2.6	0.15
$\kappa_3(M_X)$	2.5	2.5	0.5
$\kappa_{1,2}(M_X)$	0.2	2.5	0.23
s [TeV]	5.0	5.0	5.5

5.2 Results

In this section the modified particle spectrum generator is used to study the effect of the threshold corrections on the cE_6SSM parameters numerically. In this process the corrections for g_3 are studied separately from g_1 and g_2, because the latter are incomplete. Furthermore, it is expected, that the threshold corrections for g_3 have the biggest effect on the particle spectrum, because the beta function for g_3 vanishes in the E_6SSM at one-loop level (see Appendix B.9). This implies, that the running of g_3 is driven by two-loop effects only. In order to achieve a good precision for the prediction of the particle masses it is therefore necessary to handle all one-loop effects for g_3 correctly.

5.2.1 Dependency on the matching scale

In order to visualize the dependency of the model parameters on the matching scale $T_{\mathrm{E_6SSM}}$, the latter is in the following varied in the interval $[\frac{1}{2}T_0, 2T_0]$. The scale T_0 here is defined to be the renormalization scale, where the one-loop threshold corrections to g_3 vanish. As shown in the next sections, the corrections to g_3 have the biggest effect on the particle spectrum. Therefore T_0 can be interpreted as the optimal $T_{\mathrm{E_6SSM}}$ value for integrating out all new $\mathrm{E_6SSM}$ particles. The resulting variation of the model parameters with $T_{\mathrm{E_6SSM}}$ is then an estimation of the theoretical error from missing higher order contributions. In particular with this method one is able to estimate the theoretical error on the predicted particle spectrum.

5.2.2 Gauge couplings

In Figure 5.2 the dependency of gauge couplings on the matching scale $T_{\mathrm{E_6SSM}}$ is shown. The plotted values $g_i(Q)$ are the values of the couplings at a fixed scale $Q = 3\,\mathrm{TeV}$. In the case of trivial matching (5.8) one finds a clear (unphysical) dependence of $g_i(Q)$ on $T_{\mathrm{E_6SSM}}$. The slopes of $g_i(Q)(T_{\mathrm{E_6SSM}})$ are mainly given by the difference $\Delta\beta_i$ of the one-loop beta functions between the $\mathrm{E_6SSM}$ and the Standard Model

$$\Delta\beta_i := \beta_i^{\mathrm{E_6SSM}} - \beta_i^{\mathrm{SM}} \,, \tag{5.21}$$

if two-loop as well as iteration effects are small. In Table 5.2 the $\Delta\beta_i$ as well as the slopes of $g_i(Q)(T_{\mathrm{E_6SSM}})$ are listed. The values coincide with a deviation of 5 %, which can be interpreted as the mentioned higher order effects. When the threshold corrections are added, one can see a reduction in the matching scale dependency. As stated in Section 4.3.2 the change of the slopes of $g_i(Q)(T_{\mathrm{E_6SSM}})$ when adding threshold corrections is given by the sum c_i of all prefactors of the logarithms in Eq. (5.9)–(5.11). Formally the c_i are determined by taking the derivative of Eq. (5.9)–(5.11) with respect to $t \equiv \log T_{\mathrm{E_6SSM}}$

$$c_i = \frac{(4\pi)^2}{g_i^3}\frac{\mathrm{d}}{\mathrm{d}t}\left(g_i^{\overline{\mathrm{DR}},\mathrm{E_6SSM}} - g_i^{\overline{\mathrm{MS}},\mathrm{SM}}\right) \,. \tag{5.22}$$

The values of the slopes in Figure 5.2 with implemented threshold corrections as well as the coefficients c_i are listed in Table 5.2. One finds an agreement between the theoretical one-loop slope $(\Delta\beta_i - c_i)$ and the slope numerically obtained from the program, with a deviation of about 5 %. The latter mainly results from higher order corrections and effects from the iteration.

5.2.3 Particle masses

The implemented threshold corrections for the gauge couplings directly affect the particle masses, because the renormalization group equations for the soft mass parameters depend on g_i [1]. Iterative effects can also play a role, because a change in

Table 5.2: Effect of the threshold corrections on the dependency of the gauge couplings on $T_{\mathrm{E_6SSM}}$ for parameter point PP1 (see Figure 5.2). The quantities $\Delta\beta_i$ and c_i are defined in Eq. (5.21) and (5.22). The slopes of $g_i(Q)(T_{\mathrm{E_6SSM}})$ in the last three columns are obtained by linear fits to the data in Figure 5.2.

				Absolute slopes of $g_i(Q)$ when $T_{\mathrm{E_6SSM}}$ is varied		
Coupling	$\Delta\beta_i$	c_i	$\Delta\beta_i - c_i$	w/o thresh.	w/ thresh. g_3	w/ thresh. g_1, g_2, g_3
g_1	9.17	7.33	1.84	9.46	9.43	2.06
g_2	7.17	4	3.17	7.46	7.44	3.37
g_3	7	7	0	7.32	0.38	0.45

the gauge couplings at $T_{\mathrm{E_6SSM}}$ changes the value of the unification scale M_X and thus affects the values of the soft masses at low energies.

In Figures 5.3–5.4 the dependency of the particle masses on the matching scale for parameter point PP1 is shown for the gluino, the lightest neutralino and two exotic quarks. If no threshold corrections are used, one can see a change of the masses with $T_{\mathrm{E_6SSM}}$ in the range of 7–57 % (the percentage value is defined as the full variation of the particle mass divided by the mean value). The neutralino and the gluino here show the biggest dependency with 57 % and 35 %, respectively.

In case of implemented threshold corrections, one finds a reduction of the variation down to 0.6–15 % for these particles. The biggest decrease is found for the gluino, where the mass variation is reduced from 35 % to 1 %. This is because the gluino mass is very sensitive to g_3, as can be seen from the RGEs of the soft parameter M_3 [1]. The error on the neutralino is reduced from 57 % down to 15 %. This suggests that there are still further big contributions missing for this particle. One also finds, that the partial corrections for g_1 and g_2 according to Eq. (5.9)–(5.10) only have a minor effect on the masses for this parameter point. They lead to a further change in the mass variation of about 1–2 %, compared to the values with implemented corrections for g_3. Note, that the variation of the particle masses is also influenced by iterative effects, which can be of the order of up to 5 %. Therefore, the 1–2 % error reduction, that was found when adding threshold corrections for g_1 and g_2, is a mixture of both the stabilization of the gauge couplings and iteration effects.

In Figure 5.5 and 5.6 extracts of the generated particle spectra for the parameter points PP1 and PP2 are shown. The variation of the particle masses is drawn with a white box in case of trivial matching and with a black box in case of implemented threshold corrections. In the upper plots only corrections to g_3 are used and in the lower plots all corrections from Eq. (5.9)–(5.11) are implemented. When considering only g_3 corrections, the error for most of the particles gets typically reduced by about a factor one half, except for the gluino and the exotic quarks. The latter are more sensitive to g_3 and therefore show a bigger decrease in the error, as discussed above. The additional corrections to g_1 and g_2 lead to a further reduction of the matching scale dependency for most of the particle masses by 1–5 %. However, for the gluino and the exotic quark x_1 the error is increased by 1–2 %. To demonstrate that this effect can be much bigger, parameter point PP3 was constructed, where the threshold corrections to the gauge couplings spoil the matching scale dependency for some of the particle masses.

An uncommon parameter point

The parameter point PP3 shows a special behavior when threshold corrections for the couplings are added. In Figure 5.7 the effect of the corrections to the gauge couplings is displayed. Analog to PP1 one finds the predicted reduction of the dependency of $g_i(Q)$ on the matching scale with 5 % deviation. The effect of the threshold corrections on the particle masses is shown in Figure 5.8–5.10. For the gluino, the lightest neutralino and the exotic fermion x_1 the corrections lead to a reduction of the mass variation with $T_{\mathrm{E_6SSM}}$ by 10–40 %. However, the mass variation is increased for the exotic squark $\tilde{x}_{11}$ by 4 % and for the neutral first generation Higgses h^0_{11}, h^0_{21} by 10 % and 23 %. It is unlikely that such big effects come from higher order corrections only. Instead, it can be argued that the variation of the particle masses is affected by the variation of the Yukawa couplings λ_i, κ_i, f^k_{ij} due to iteration effects. If there exists an accidental cancellation between the error of the Yukawa and the error of the gauge couplings without threshold corrections, a decrement of the error of the g_i by including threshold corrections can increase the overall error on the particle masses. This may explain the increase in the scale dependence for this parameter point.

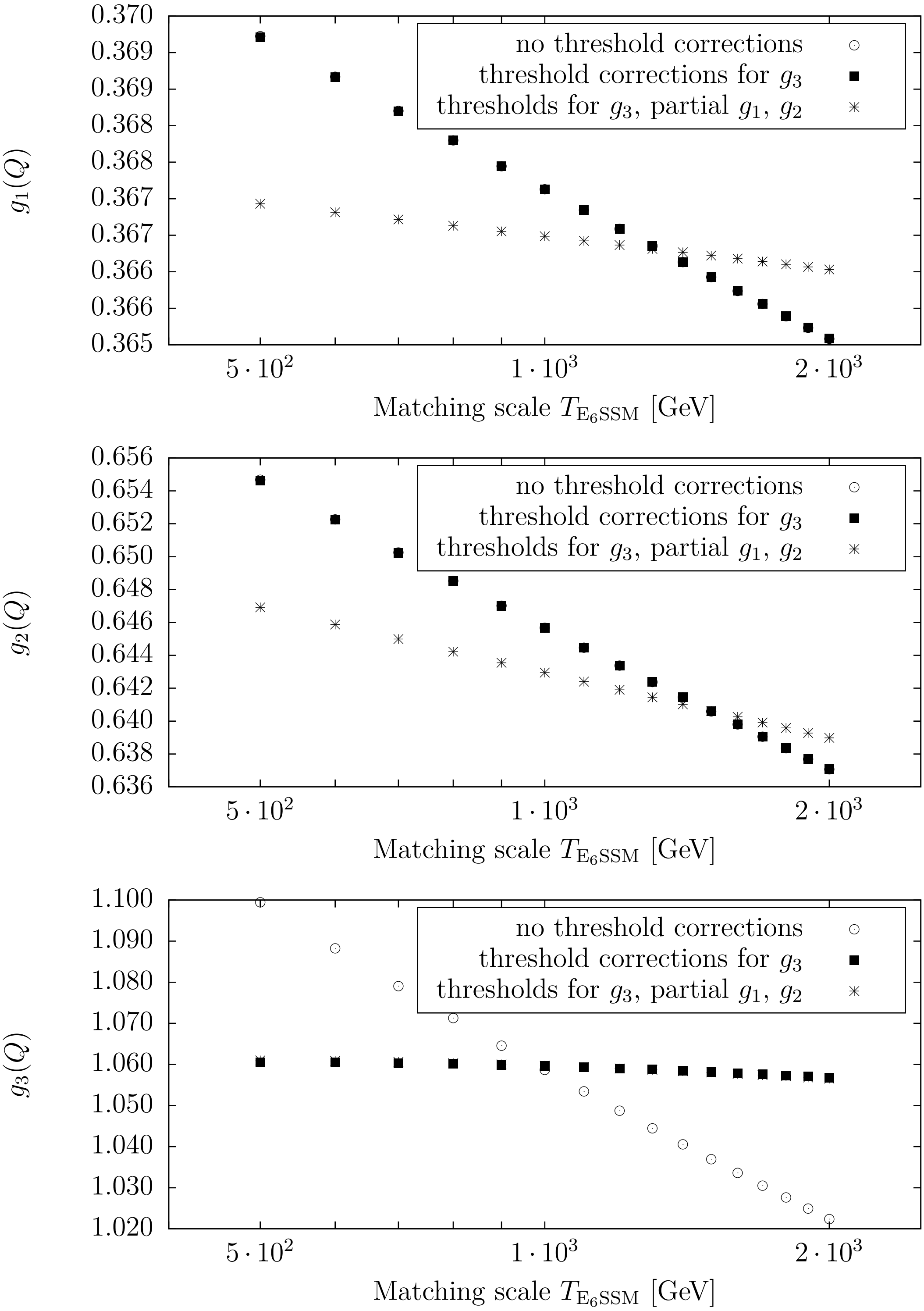


Figure 5.2: Dependency of the gauge couplings $g_i(Q)$ at $Q = 3\,\mathrm{TeV}$ on the matching scale $T_{\mathrm{E_6SSM}}$ for parameter point PP1. The circles show the behavior without threshold corrections, the squares with corrections for g_3, and the stars with corrections for g_3 and partial g_1 and g_2 according to Eq. (5.9)–(5.11). In the first and second plot the data points without thresholds and with corrections to g_3 coincide. The same happens in the last plot when the corrections for g_1 and g_2 are added to g_3.

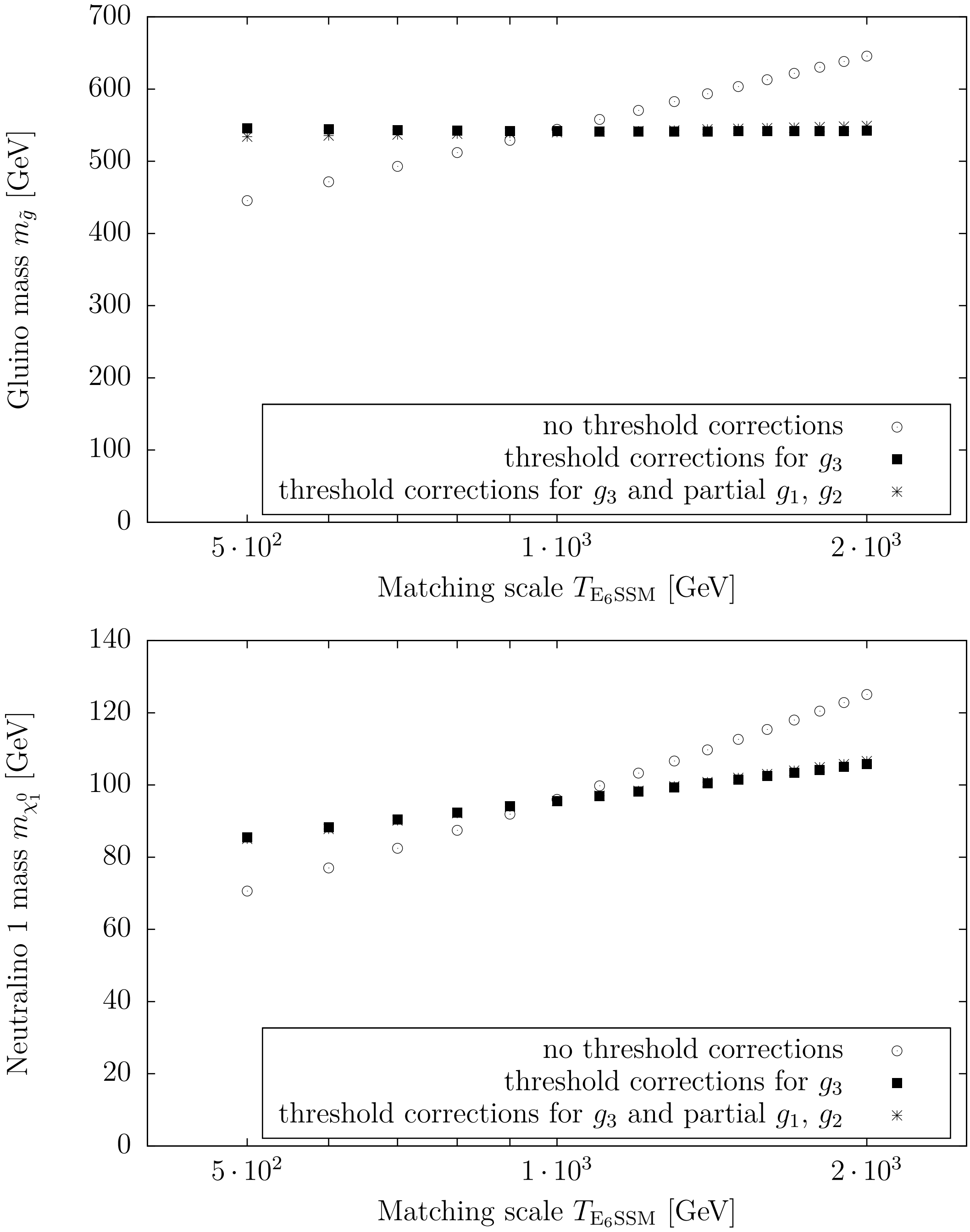


Figure 5.3: Dependency of the gluino and neutralino mass on the matching scale $T_{\mathrm{E_6SSM}}$ for parameter point PP1. The circles show the behavior without threshold corrections, the squares with corrections for g_3, and the stars with corrections for g_3 and partial g_1 and g_2 according to Eq. (5.9)–(5.11).

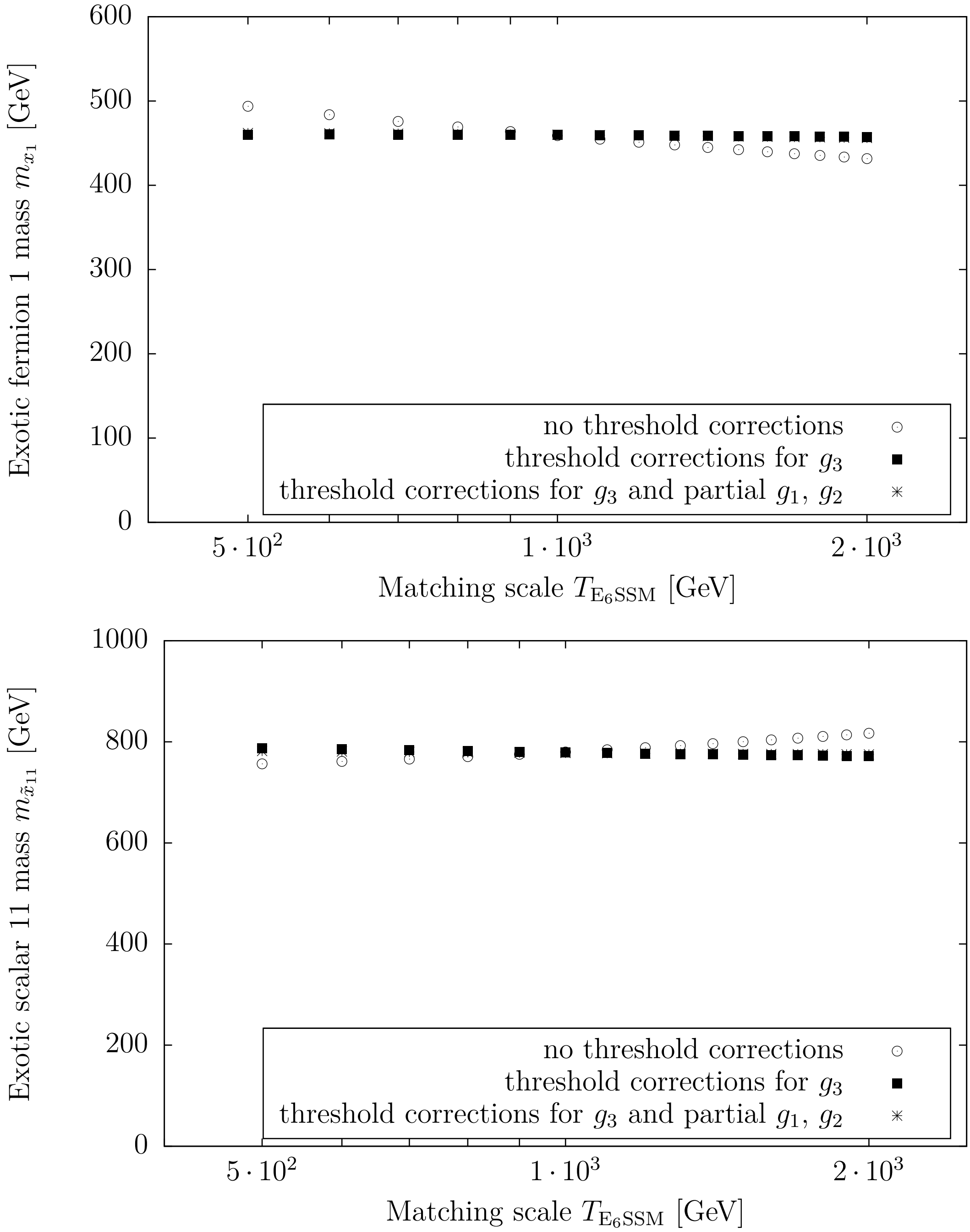


Figure 5.4: Dependency of the exotic particle's masses on the matching scale $T_{\mathrm{E_6SSM}}$ for parameter point PP1. The circles show the behavior without threshold corrections, the squares with corrections for g_3, and the stars with corrections for g_3 and partial g_1 and g_2 according to Eq. (5.9)–(5.11). In the first and second plot the data points with threshold corrections to g_3 and with corrections to g_1, g_2, g_3 almost coincide.

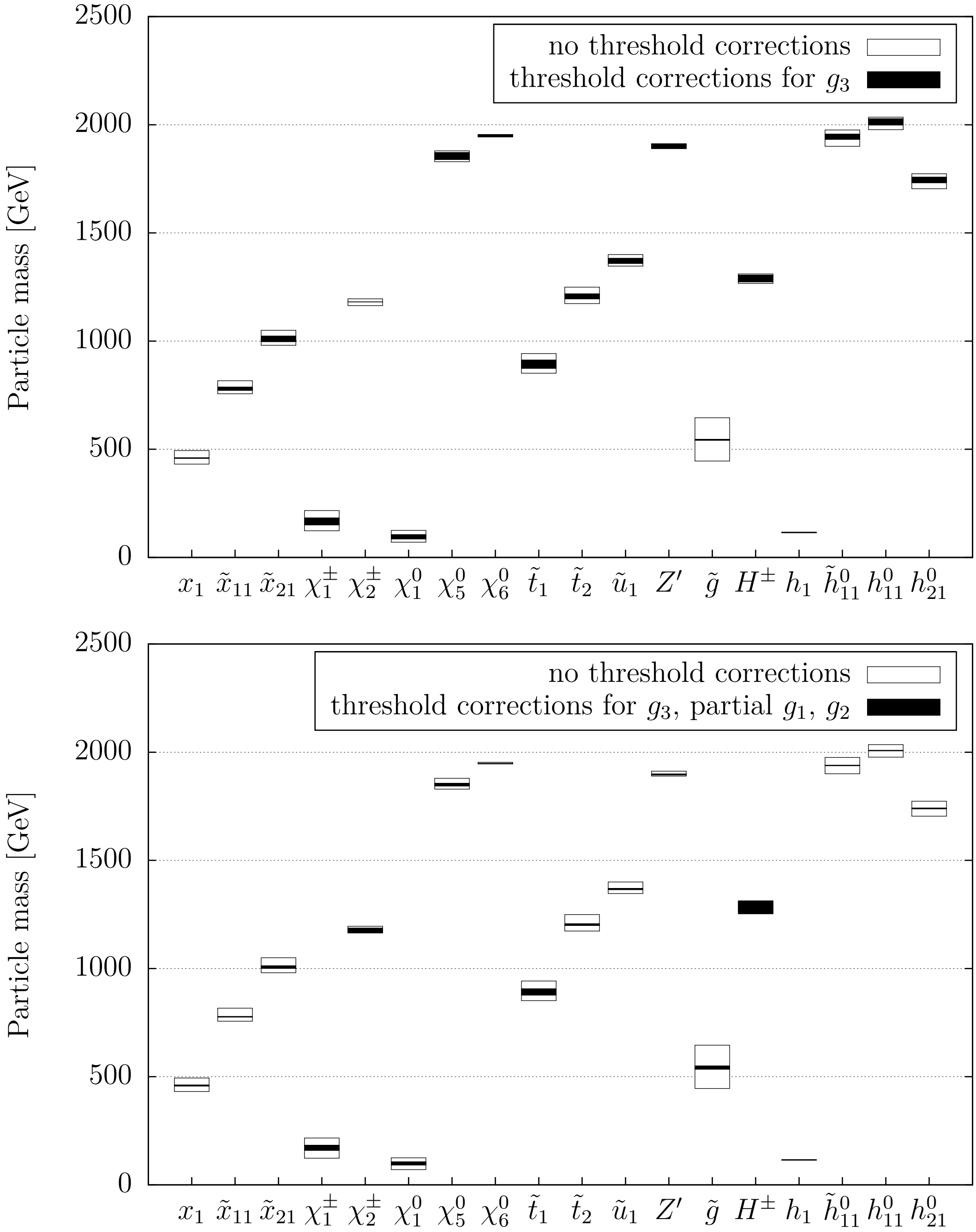


Figure 5.5: Particle spectra for parameter point PP1. The white and the black boxes show the variation of the particle masses when $T_{\mathrm{E_6SSM}}$ is varied in the interval $[\frac{1}{2}T_0, 2T_0]$. The black boxes show the error with threshold corrections and the white boxes without.

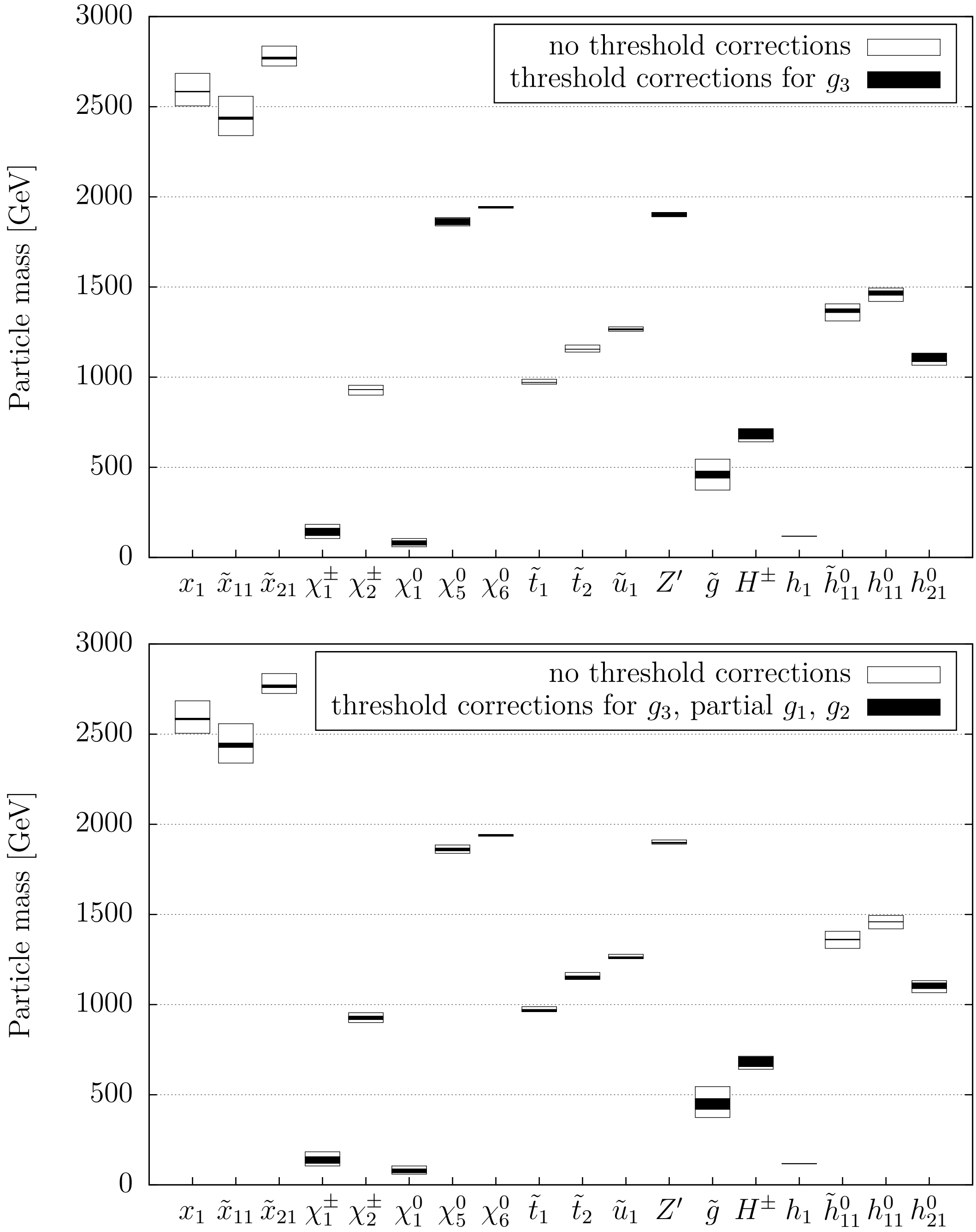


Figure 5.6: Particle spectra for parameter point PP2. The white and the black boxes show the variation of the particle masses when $T_{\mathrm{E_6SSM}}$ is varied in the interval $[\frac{1}{2}T_0, 2T_0]$. The black boxes show the error with threshold corrections and the white boxes without.

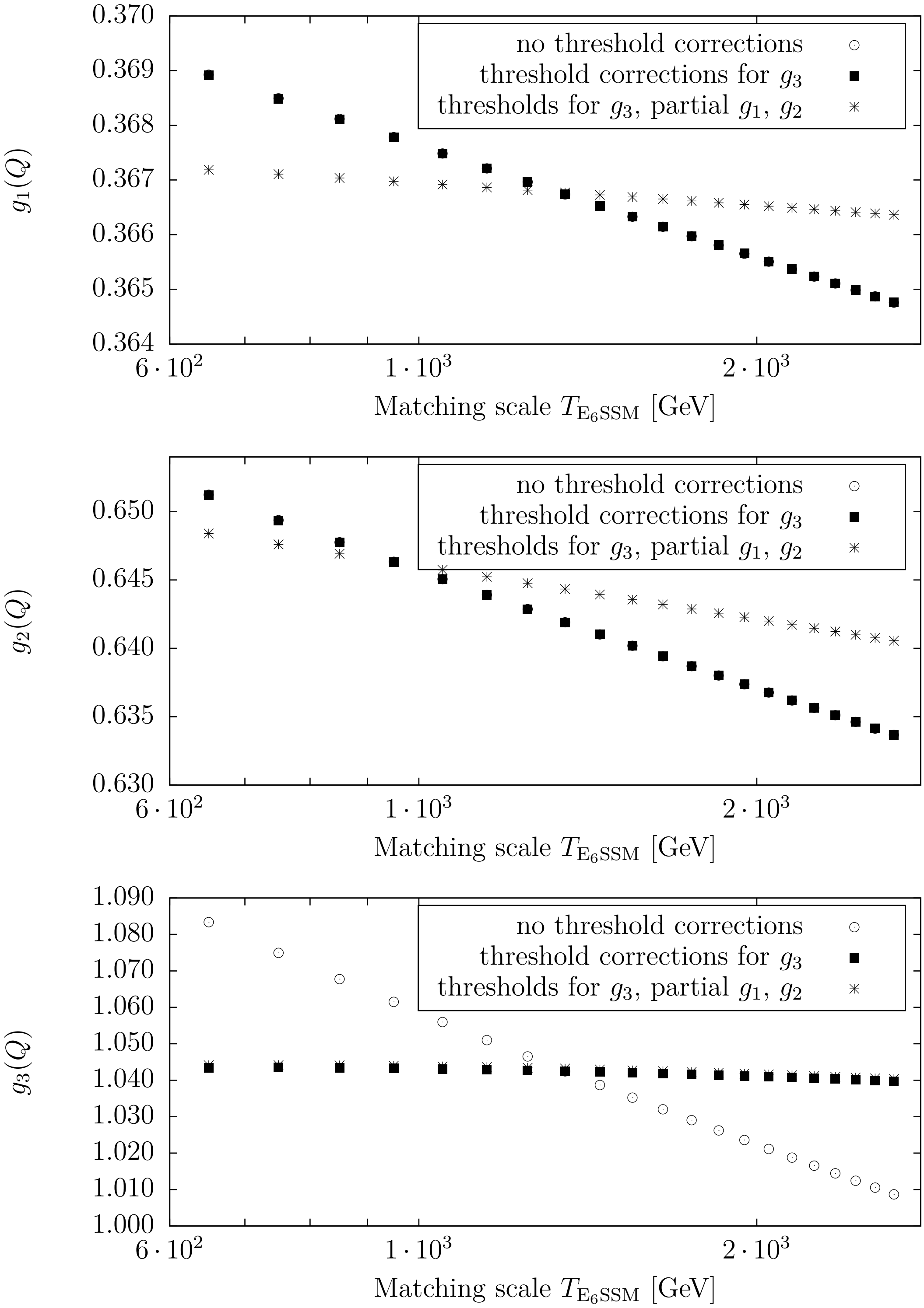


Figure 5.7: Dependency of the gauge couplings $g_i(Q)$ at $Q = 3\,\mathrm{TeV}$ on the matching scale $T_{\mathrm{E_6SSM}}$ for parameter point PP3. The circles show the behavior without threshold corrections, the squares with corrections for g_3, and the stars with corrections for g_3 and partial g_1 and g_2. In the first and second plot the data points without thresholds and with corrections to g_3 coincide. The same happens in the last plot when the corrections for g_1 and g_2 are added to g_3.

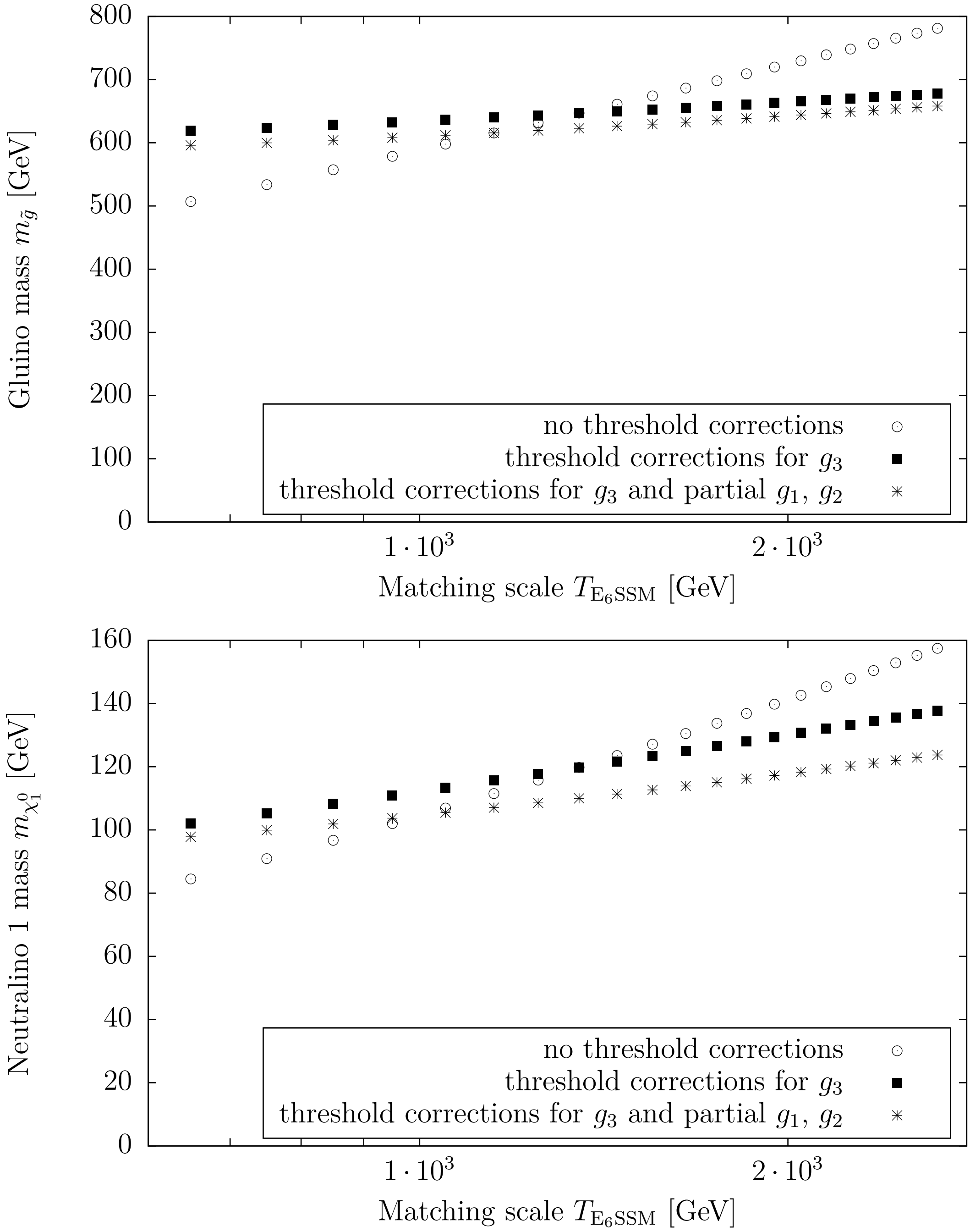


Figure 5.8: Dependency of the gluino and neutralino mass on the matching scale $T_{\mathrm{E_6SSM}}$ for parameter point PP3. The circles show the behavior without threshold corrections, the squares with corrections for g_3, and the stars with corrections for g_3 and partial g_1 and g_2 according to Eq. (5.9)–(5.11).

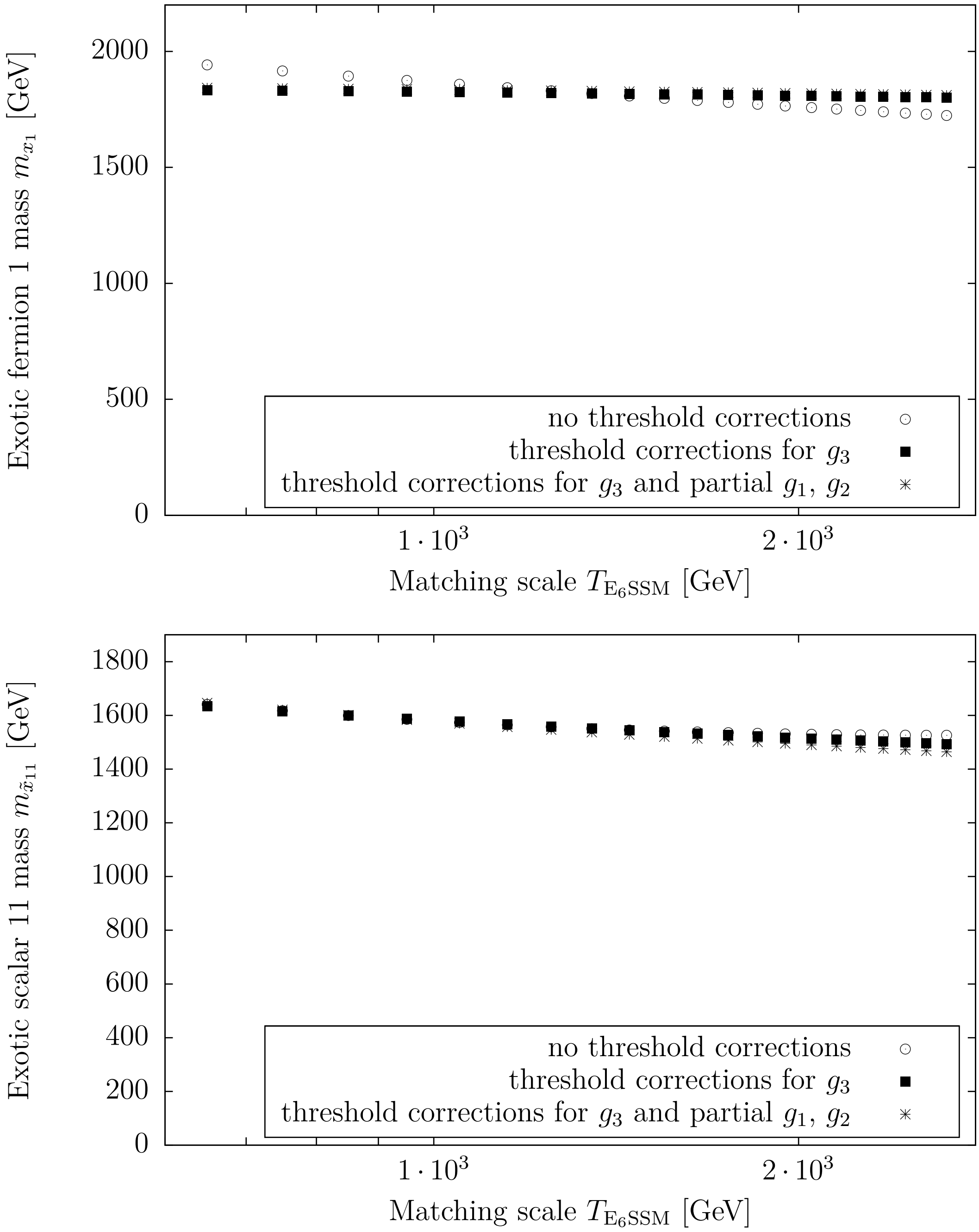


Figure 5.9: Dependency of the exotic particle's masses on the matching scale $T_{\mathrm{E_6SSM}}$ for parameter point PP3. The circles show the behavior without threshold corrections, the squares with corrections for g_3, and the stars with corrections for g_3 and partial g_1 and g_2 according to Eq. (5.9)–(5.11).

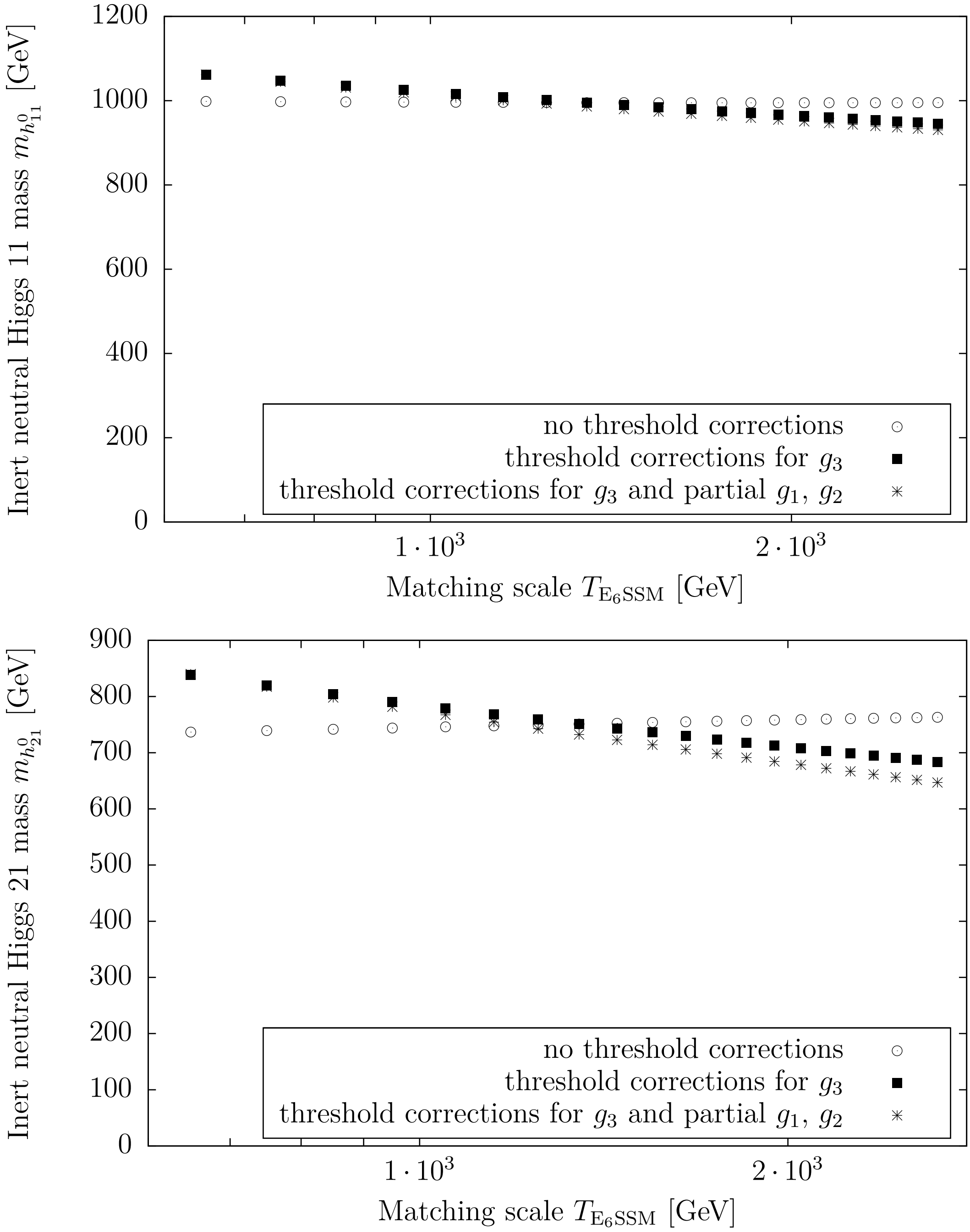


Figure 5.10: Dependency of the neutral first generation Higgs masses on the matching scale T_{E_6SSM} for parameter point PP3. The circles show the behavior without threshold corrections, the squares with corrections for g_3, and the stars with corrections for g_3 and partial g_1 and g_2 according to Eq. (5.9)–(5.11).

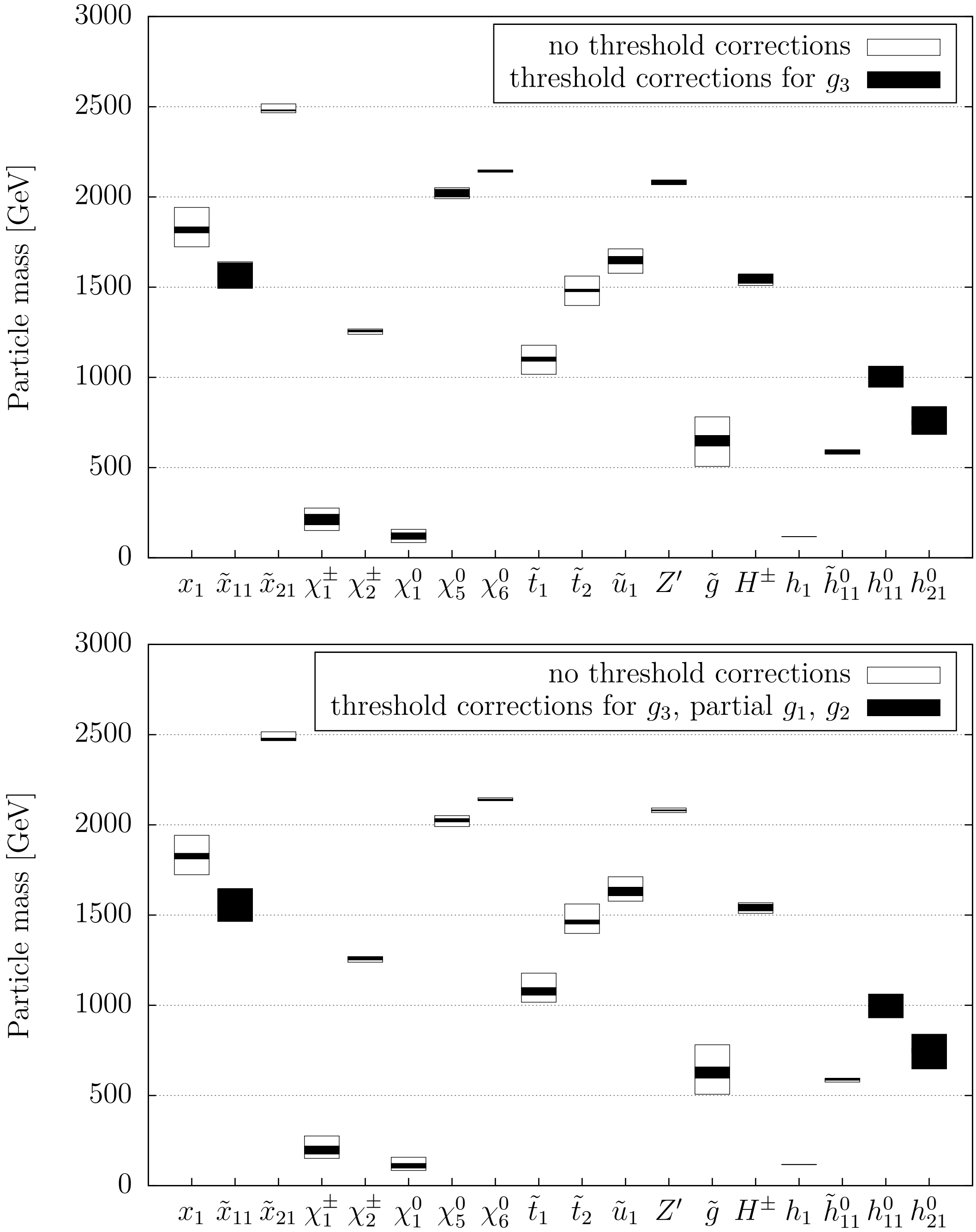


Figure 5.11: Particle spectrum for parameter point PP3. The white and the black boxes show the variation of the particle masses when $T_{\mathrm{E_6SSM}}$ is varied in the interval $[\frac{1}{2}T_0, 2T_0]$. The white boxes show the error without threshold corrections and the black boxes with implemented corrections for g_3.

6 Summary and outlook

The constrained Exceptional Supersymmetric Standard Model (cE_6SSM) is an attractive supersymmetric model for physics beyond the Standard Model. It is inspired by E_6 grand unified theories which can be constructed from heterotic string theory based on $E_8 \times E_8'$. The latter can provide a unified description of the strong, electroweak and gravitational interaction as well as the framework of a hidden sector to dynamically break supersymmetry at lower energies.

In 2009 first cE_6SSM particle spectra were calculated [1], using a spectrum generator based on `SOFTSUSY` 2.0.5. However, the calculation procedure neglects threshold corrections to renormalization group running of the model parameters. This results in an unphysical dependency of the predicted spectrum on the arbitrary scale where the cE_6SSM is matched to the Standard Model. As a consequence an inaccuracy of the predicted particle masses of the order 10–50 % exists.

In this thesis one-loop threshold corrections to the gauge couplings were calculated, in order to improve the precision of the predicted cE_6SSM particle spectrum. The results were found to be in agreement with an earlier calculation done by Hall [44] for a general broken gauge theory. The implementation of the cE_6SSM threshold corrections into the particle spectrum generator showed a significant reduction of the error on the calculated masses for most of the particles. However, parameter points were found where the incorporation of the threshold corrections to the gauge couplings leads to an increase of the error on the calculated particle masses. It was argued that such effects could appear if there is an accidental cancellation between the errors of the gauge and Yukawa couplings, if no threshold corrections are used.

In a following Ph. D. thesis the calculation of the threshold corrections to the gauge couplings will be studied in more detail to also account for (s)particle mixing. An analogue calculation will be done for the cE_6SSM Yukawa couplings f_{ij}^k to study the above mentioned cancellation and to reduce the error on the predicted particle masses further. It is also planed to check the validity of the cE_6SSM by calculating radiative corrections from cE_6SSM particles to electroweak precision observables.

A Notations and definitions

A.1 Conventions

In the text natural units are used, $c = \hbar = 1$. The metric tensor is written in the time-like sign convention

$$g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1) \; . \tag{A.1}$$

To handle vector boson 2-point functions, it is convenient to introduce the transverse projector $\mathrm{P}_{\mathrm{T}}^{\mu\nu}$ by

$$\mathrm{P}_{\mathrm{T}}^{\mu\nu} := g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \tag{A.2}$$

so that the transverse parts of a correlation function can be written as

$$\Gamma_{V_\mu^a V_\nu^b,\mathrm{T}} = \mathrm{P}_{\mathrm{T}}^{\mu\nu} \, \Gamma_{V_\mu^a V_\nu^b} \; . \tag{A.3}$$

Note, that the transverse part $\Gamma_{V_\mu^a V_\nu^b,\mathrm{T}}$ has no free Lorentz indices.

A.2 The path integral

The aim of the path integral formalism is to express transition amplitudes $\langle x_f, t_f | x_i, t_i \rangle$ as an integral over a function space. Calculating this amplitude is equivalent to solving the Schrödinger equation, because the time development of a wave function can then be obtained from

$$\psi(x_f, t_f) = \int \mathrm{d}x_i \, \langle x_f, t_f | x_i, t_i \rangle \, \psi(x_i, t_i) \; . \tag{A.4}$$

By splitting the time interval $[t_i, t_f]$ into equidistant pieces of length $\Delta t = (t_f - t_i)/n$ and inserting complete sets of basis states, the amplitude can be rewritten in the form

$$\langle x_f, t_f | x_i, t_i \rangle = N_n^n \int \prod_{j=1}^{n-1} \mathrm{d}x_j \; \exp\left\{ \mathrm{i}\Delta t \sum_{j=0}^{n-1} \left[\frac{m}{2} \left(\frac{x_{j+1} - x_j}{\Delta t} \right)^2 - V \right] \right\} , \tag{A.5}$$

where $N_n^n = (\frac{m}{2\pi \mathrm{i}\Delta t})^{n/2}$. Taking the limit $n \to \infty$ naively and writing the amplitude as

$$\langle x_f, t_f | x_i, t_i \rangle = \int \mathcal{D}x \ \exp\left\{\mathrm{i}\int_{t_i}^{t_f} \mathrm{d}t \ L(x, \dot{x})\right\} \tag{A.6}$$

in the sense of a functional integral is unsafe, because both N_n^n and $\int \prod_{j=1}^{n-1} \mathrm{d}x_j$ are divergent as $n \to \infty$. Furthermore it can be shown that

$$\mathcal{D}x \ \exp\left\{\mathrm{i}\int_{t_i}^{t_f} \mathrm{d}t \ L(x, \dot{x})\right\} \tag{A.7}$$

cannot define a measure in a classical mathematical sense [53]. For this reason the Feynman path integral is not an integral in the standard sense.

This problem can be avoided by going to imaginary time $\tau = \mathrm{i}t$, which makes the expression

$$N_n^n \prod_{j=1}^{n-1} \mathrm{d}x_j \ \exp\left\{-\Delta\tau \sum_{j=0}^{n-1} \left[\frac{m}{2}\left(\frac{x_{j+1} - x_j}{\Delta\tau}\right)^2\right]\right\} \tag{A.8}$$

converge to the well defined Wiener measure in the limit $n \to \infty$ [54]. In this sense Eq. (A.8) can be used as a definition of (A.7) after taking the limit $n \to \infty$ and switching back to real time. Note that, in order to guarantee the recovery of the real time path integral, certain conditions for imaginary time correlation functions have to be fulfilled, see e.g. [55].

A.3 Definition of correlation functions

In order to study an effective field theory it is convenient to consider the correlation functions of the theory. They provide the possibility of calculating relations between parameters of a quantum field theory and it's effective theory, where heavy particles are removed from the Lagrangian.

The n-point correlation function $\tau^{(n)}(x_1, \dots, x_n)$ is defined as the vacuum expectation value of a T^*-ordered operator product of causal quantum field operators

$$\tau^{(n)}(x_1, \dots, x_n) = \langle 0 | T^* \phi(x_1) \cdots \phi(x_n) | 0 \rangle \ . \tag{A.9}$$

Using the path integral representation of transition amplitudes, one can rewrite Eq. (A.9) as

$$\tau^{(n)}(x_1, \dots, x_n) = N \int \mathcal{D}\phi \ \phi(x_1) \cdots \phi(x_n) \ \exp\left(\mathrm{i}\int \mathrm{d}^4x \ \mathcal{L}\right) \tag{A.10}$$

$$= (-\mathrm{i})^n \left. \frac{\delta^n Z[J]}{\delta J(x_1) \cdots \delta J(x_n)} \right|_{J=0} \tag{A.11}$$

where

$$Z[J] = N \int \mathcal{D}\phi \;\exp\Big(\mathrm{i}\int \mathrm{d}^4x\;(\mathcal{L} + J\phi)\Big) \tag{A.12}$$

$$N^{-1} = \int \mathcal{D}\phi \;\exp\Big(\mathrm{i}\int \mathrm{d}^4x\;\mathcal{L}\Big)\;. \tag{A.13}$$

In a perturbation expansion of Eq. (A.10) one will notice that the full correlation function contains many terms which are products of unconnected correlations functions of lower order. It is therefore convenient to introduce the generator $W[J]$ for the connected correlation functions

$$W[J] = -\mathrm{i}\log Z[J]\;. \tag{A.14}$$

The connected correlation functions can now be obtained via

$$\tau_c^{(n)}(x_1,\dots,x_n) = (-\mathrm{i})^{n-1}\;\frac{\delta^n W[J]}{\delta J(x_1)\cdots\delta J(x_n)}\bigg|_{J=0}\;. \tag{A.15}$$

One can reduce the connected correlation functions even further to contain only graphs which do not break into disconnected parts if a single internal propagator is removed. Those correlation functions are called one particle irreducible (1PI). The generator Γ of these functions is obtained from a Legendre transformation on $W[J]$

$$\Gamma[\phi_c] = W[J] - \int \mathrm{d}^4x\; J(x)\phi_c(x)\;, \qquad \text{where} \qquad \phi_c(x) = \frac{\delta W[J]}{\delta J(x)}\;. \tag{A.16}$$

Analogous to Eq. (A.15) all one particle irreducible n-point correlation functions are now given by

$$\Gamma^{(n)}(x_1,\dots,x_n) = \frac{\delta^n \Gamma[\phi_c]}{\delta\phi_c(x_1)\cdots\delta\phi_c(x_n)}\bigg|_{\phi_c=0}\;. \tag{A.17}$$

By making use of Eq. (A.14), $\Gamma[\phi_c]$ can again be expressed in terms of a path integral

$$\Gamma[\phi_c] = -\mathrm{i}\log N \int \mathcal{D}\phi\;\exp\Big(\mathrm{i}\int \mathrm{d}^4x\;\big[\mathcal{L}(\phi) + J(\phi-\phi_c)\big]\Big) \tag{A.18}$$

$$= -\mathrm{i}\log N \int \mathcal{D}\hat{\phi}\;\exp\Big(\mathrm{i}\int \mathrm{d}^4x\;\big[\mathcal{L}(\phi_c+\hat{\phi}) + J\hat{\phi}\big]\Big)\;. \tag{A.19}$$

A.4 Superfield formalism

Throughout the text, the notation of [22, 23] is used.

A.4.1 Definition of Grassmann variables

Grassmann variables θ_i are elements of an algebra with the inner product

$$\{\theta_i, \theta_j\} := \theta_i\theta_j + \theta_j\theta_i = 0 \tag{A.20}$$

The algebra can be enlarged by an involution $\mathcal{I}$

$$\mathcal{I} : \theta_i \to \mathcal{I}(\theta_i) \equiv \bar{\theta}_i \ , \qquad \mathcal{I}(\mathcal{I}(\theta_i)) = \theta_i \tag{A.21}$$

where the conjugate Grassmann variables $\bar{\theta}_i$ obey

$$\{\bar{\theta}_i, \bar{\theta}_j\} = 0 \qquad \text{and} \qquad \{\theta_i, \bar{\theta}_j\} = 0 \ . \tag{A.22}$$

Because of Eq. (A.20) and (A.21) every function depending on Grassmann variables can be expanded into

$$f(\theta_i, \bar{\theta}_j) = a_0 + \sum_i a_i\theta_i + \sum_i \bar{a}_i\bar{\theta}_i + \cdots \ . \tag{A.23}$$

Furthermore one can define differentiation and integration on a Grassmann algebra via

$$\frac{\partial}{\partial\theta_i}\theta_j := \delta_{ij} \tag{A.24}$$

$$\frac{\partial}{\partial\theta_i}\theta_j\theta_k := \delta_{ij}\theta_k - \delta_{ik}\theta_j \tag{A.25}$$

$$\int \mathrm{d}\theta_i := 0 \tag{A.26}$$

$$\int \mathrm{d}\theta_i \ \theta_j := \delta_{ij} \tag{A.27}$$

with $\{\mathrm{d}\theta_i, \mathrm{d}\theta_j\} = 0$.

A.4.2 Two component notation

To formulate supersymmetry in an elegant way it is most convenient to introduce a two component decomposition of a Dirac spinor ψ in terms of Weyl spinors

$$\psi_a = \begin{pmatrix} \xi_A \\ \bar{\chi}^{\dot{A}} \end{pmatrix} \ , \qquad (a = 1, 2, 3, 4), \ (A = 1, 2), \ (\dot{A} = \dot{1}, \dot{2}) \tag{A.28}$$

because the component spinors ξ_A, $\bar{\chi}^{\dot{A}}$ transform as fundamental representations $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ of the Lorentz group respectively. For these spinors the following relations hold

$$\xi_A = (\bar{\xi}_{\dot{A}})^\dagger \ , \qquad \xi^A = (\bar{\xi}^{\dot{A}})^\dagger \ , \tag{A.29}$$

$$\xi^A = \epsilon^{AB}\xi_B \ , \qquad \xi_A = \epsilon_{AB}\xi^B \ , \tag{A.30}$$

$$\bar{\chi}^{\dot{A}} = \epsilon^{\dot{A}\dot{B}}\bar{\chi}_{\dot{B}} \;, \qquad \bar{\chi}_{\dot{A}} = \epsilon_{\dot{A}\dot{B}}\bar{\chi}^{\dot{B}} \;, \tag{A.31}$$

where the antisymmetric tensors ϵ^{AB} and ϵ_{AB} are given by

$$\epsilon^{AB} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \;, \qquad \epsilon_{AB} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{A.32}$$

and it holds that $\epsilon^{AB}\epsilon_{BC} = \delta^A_C$. Furthermore one defines, that the spinor components ξ_A and $\bar{\chi}^{\dot{A}}$ are Grassmann variables, i. e.,

$$\{\xi_A, \xi_B\} = \{\bar{\chi}^{\dot{A}}, \bar{\chi}^{\dot{B}}\} = \{\xi_A, \bar{\chi}^{\dot{B}}\} = 0 \;. \tag{A.33}$$

One now constructs a $SL(2, C)$ invariant spinor product

$$\xi\chi := \xi^A \chi_A \;, \tag{A.34}$$

$$\bar{\chi}\bar{\xi} := \bar{\chi}_{\dot{A}}\bar{\xi}^{\dot{A}} = (\xi\chi)^\dagger \;, \tag{A.35}$$

which is symmetric

$$\xi\chi = \chi\xi = (\bar{\xi}\bar{\chi})^\dagger = (\bar{\chi}\bar{\xi})^\dagger \tag{A.36}$$

because of Eq. (A.33) the antisymmetry of ϵ_{AB}. One furthermore defines differential operators as follows

$$\partial_A := \frac{\partial}{\partial\theta^A} \;, \qquad \partial^A := \frac{\partial}{\partial\theta_A} \;, \qquad \bar{\partial}^{\dot{A}} := \frac{\partial}{\partial\bar{\theta}_{\dot{A}}} \;, \qquad \bar{\partial}_{\dot{A}} := \frac{\partial}{\partial\bar{\theta}^{\dot{A}}} \tag{A.37}$$

with the anti-commutation relations

$$\{\partial_A, \partial_B\} = \{\bar{\partial}_{\dot{A}}, \bar{\partial}_{\dot{B}}\} = \{\partial_A, \bar{\partial}_{\dot{B}}\} = 0 \;. \tag{A.38}$$

Per definition they act on the Grassmann variables via

$$\partial_A\theta^B := \delta^B_A \;, \qquad \partial^A\theta_B := \delta^A_B \;, \qquad \bar{\partial}_{\dot{A}}\bar{\theta}^{\dot{B}} := \delta^{\dot{B}}_{\dot{A}} \;, \qquad \bar{\partial}^{\dot{A}}\bar{\theta}_{\dot{B}} := \delta^{\dot{A}}_{\dot{B}} \tag{A.39}$$

$$\partial^A\theta^B := -\epsilon^{AB} \;, \qquad \partial_A\theta_B := -\epsilon_{AB} \;, \qquad \bar{\partial}_{\dot{A}}\bar{\theta}_{\dot{B}} := -\epsilon_{\dot{A}\dot{B}} \;, \qquad \bar{\partial}^{\dot{A}}\bar{\theta}^{\dot{B}} := -\epsilon^{\dot{A}\dot{B}} \tag{A.40}$$

and transform as

$$\epsilon^{AB}\partial_B = -\partial^A \;, \quad \epsilon_{AB}\partial^B = -\partial_A \;, \quad \epsilon_{\dot{A}\dot{B}}\bar{\partial}^{\dot{B}} = -\bar{\partial}_{\dot{A}} \;, \quad \epsilon^{\dot{A}\dot{B}}\bar{\partial}_{\dot{B}} = -\bar{\partial}^{\dot{A}} \;. \tag{A.41}$$

Furthermore the shorthands

$$\mathrm{d}^2\theta \;:= -\frac{1}{4}\mathrm{d}\theta^A\mathrm{d}\theta_A \;, \qquad \mathrm{d}^2\bar{\theta} \;:= -\frac{1}{4}\mathrm{d}\bar{\theta}_{\dot{A}}\mathrm{d}\bar{\theta}^{\dot{A}} \;, \qquad \mathrm{d}^4\theta \;:= \mathrm{d}^2\theta\,\mathrm{d}^2\bar{\theta} \tag{A.42}$$

are introduced in order to construct projectors for components of the superfields (3.7). One can then write

$$\int \mathrm{d}^2\theta\ \mathcal{F}(z) = M(x)\ , \tag{A.43}$$

$$\int \mathrm{d}^2\bar{\theta}\ \mathcal{F}(z) = N(x)\ , \tag{A.44}$$

$$\int \mathrm{d}^4\theta\ \mathcal{F}(z) = \frac{1}{2}D(x)\ . \tag{A.45}$$

B E_6 grand unified theory

B.1 Definition of Lie group and Lie algebra

In this section the definition of Lie group and Lie algebra is given.

Definition 1. A group $(G, \circ)$ is a set G together with a binary operation $\circ$ that obey

1. $g, f \in G \Rightarrow g \circ f \in G$
2. $(g \circ f) \circ h = g \circ (f \circ h)\ \forall g, f, h \in G$
3. $\exists e \in G : e \circ g = g \circ e = g\ \forall g \in G$
4. $\forall g \in G\ \exists g^{-1} \in G : g \circ g^{-1} = g^{-1} \circ g = e$

Definition 2. Let $(G, \circ)$ be a group.

1. G is called abelian, if $g \circ f = f \circ g\ \forall g, f \in G$
2. A subset $A \subset G$ is called subgroup, if $(A, \circ)$ is a group.
3. A subgroup $A \subset G$ is called invariant subgroup, if

$$g \circ a_i \circ g^{-1} = a_j \qquad \forall g \in G, \quad \forall a_i, a_j \in A\ . \tag{B.1}$$

Definition 3. A group G is called Lie group, if its elements g are continuous functions of one or more continuous parameters ρ_a.

Definition 4. Let G be a Lie group.

1. G is called compact, if the parameter space of G, spanned by the ρ_a, is compact.
2. G is called simple, if no invariant Lie subgroup $A \subset G$ exists.
3. G is called semi-simple, if no invariant abelian Lie subgroup $A \subset G$ exists.

Proposition 1. Let G be a Lie group with n parameters ρ_a. The elements $U(\rho_1, \ldots, \rho_n)$ of G can be expressed in the form

$$U(\rho_1, \ldots, \rho_n) = \exp\left(-\mathrm{i} \sum_{a=1}^{n} \rho_a T^a\right)\ , \tag{B.2}$$

where T^a are called generators of the group.

Definition 5. A field $(\mathbb{K}, +, \cdot)$ is a set $\mathbb{K}$ together with two binary operations

$$+ : \mathbb{K} \times \mathbb{K} \to \mathbb{K} \tag{B.3}$$
$$\cdot : \mathbb{K} \times \mathbb{K} \to \mathbb{K} \tag{B.4}$$

so that $(\mathbb{K}, +)$ and $(\mathbb{K}, \cdot)$ are both abelian groups and furthermore it holds that

$$a \cdot (b + c) = a \cdot b + a \cdot c \qquad \forall a, b, c \in \mathbb{K}. \tag{B.5}$$

Definition 6. A vector space $(V, \oplus, \otimes)$ over a field $(\mathbb{K}, +, \cdot)$ is a set V with two binary operations

$$\oplus : V \times V \to V \tag{B.6}$$
$$\otimes : \mathbb{K} \times V \to V \tag{B.7}$$

so that $(V, \oplus)$ is an abelian group and furthermore it holds that

1. $a \otimes (u \oplus v) = a \otimes u \oplus a \otimes v \; \forall a \in \mathbb{K} \; \forall u, v \in V$
2. $(a + b) \otimes u = a \otimes u \oplus b \otimes u \; \forall a, b \in \mathbb{K} \; \forall u \in V$
3. $a \otimes (b \otimes u) = (a \cdot b) \otimes u \; \forall a, b \in \mathbb{K} \; \forall u \in V$
4. $1 \otimes u = u \; \forall u \in V$ and $1 \in \mathbb{K}$ is the multiplicative identity in $\mathbb{K}$

Definition 7. An algebra $\mathfrak{g}$ is a vector space $(V, \oplus, \otimes)$ over a field $(\mathbb{K}, +, \cdot)$ equipped with an additional binary operation

$$\bullet : V \times V \to V \tag{B.8}$$

for which the following holds

1. $(g \oplus f) \bullet h = g \bullet h \oplus f \bullet h \; \forall f, g, h \in V$
2. $g \bullet (f \oplus h) = g \bullet f \oplus g \bullet h \; \forall f, g, h \in V$
3. $(a \otimes g) \bullet (b \otimes f) = (a \cdot b) \otimes (g \bullet f) \; \forall a, b \in \mathbb{K} \; \forall g, f \in V$

Proposition 2. Let G be a Lie group with n parameters ρ_a. The generators of G are then given by

$$T^a = \mathrm{i} \left. \frac{\partial U(\rho_1, \ldots, \rho_n)}{\partial \rho_a} \right|_{\rho=0} \tag{B.9}$$

and they form a closed algebra with the inner product $\bullet = [,]$ defined by

$$[T^a, T^b] \equiv T^a T^b - T^b T^a = \mathrm{i} f^{abc} T^c \; . \tag{B.10}$$

This algebra is called Lie algebra $\mathfrak{g}$ of the Lie group G. The f^{abc} are called structure constants of $\mathfrak{g}$.

B.2 Gauge transformations as Lie group

In a gauge theory, the local gauge transformations $U(x)$ form a Lie group G with the associated Lie algebra $\mathfrak{g}$. In case of a one-parameter Lie group the elements $U(x)$ can be written as

$$U(x) = \exp\left(-\mathrm{i}g\lambda^a(x)T^a\right) \tag{B.11}$$

where g is the group (gauge) parameter, $\lambda^a(x)$ are called gauge functions and T^a are the generators of the group. For the latter Eq. (B.10) holds. The group factor C is defined by

$$\mathrm{Tr}[T^aT^b] = C\,\delta^{ab} \tag{B.12}$$

in a given representation of the gauge group. For $SU(N)$ groups in the fundamental representation one has $C = 1/2$ and in the adjoint representation $C = N$, where the adjoint representation is obtained by setting $(T^a)^{bc} = -\mathrm{i}f^{abc}$. Within the text the notation C_f, C_s and C_v is used for the group factors of fermions, scalars and vector bosons, respectively.

B.3 Root system of a semi-simple Lie algebra

Consider a semi-simple Lie algebra $\mathfrak{g}$ with N elements X_μ and the inner product

$$[X_\mu, X_\nu] = f_{\mu\nu\sigma}X_\sigma\ . \tag{B.13}$$

One can construct linear combinations A and X

$$A = a^\mu X_\mu\ , \qquad X = x^\mu X_\mu \tag{B.14}$$

such that an eigenvalue equation

$$[A, X] = rX \tag{B.15}$$

is fulfilled and only solutions to $r = 0$ are degenerate. The eigenvalues $r \neq 0$ are called roots of the Lie algebra. If the solutions to $r = 0$ is l times degenerate, l is called rank of the Lie algebra. The l eigenvectors to $r = 0$ are labeled H_i and the remaining $(N - l)$ eigenvectors to $r \neq 0$ are labeled E_α

$$[A, H_i] = 0 \qquad (i = 1, \ldots, l) \tag{B.16}$$

$$[A, E_\alpha] = \alpha E_\alpha\ . \tag{B.17}$$

The eigenvectors H_i and E_α span $\mathfrak{g}$. It can be shown [56] that

$$[H_i, H_j] = 0 \qquad \text{and} \qquad [H_i, E_\alpha] = \alpha_i E_\alpha\ , \tag{B.18}$$

where α_i are initially arbitrary constants. Since $[A, H_i] = 0$ and $[H_i, H_j] = 0$, A can be expressed by

$$A = a^i H_i \ . \qquad \text{(B.19)}$$

From the Jacobi identity for the commutator and equations (B.16) and (B.17) it follows that

$$[A, [H_i, E_\alpha]] = \alpha [H_i, E_\alpha] \qquad \text{(B.20)}$$

and therefore

$$\alpha = a^i \alpha_i \ . \qquad \text{(B.21)}$$

The α_i can now be seen to be components of a vector $\vec{\alpha}$ of an l dimensional vector space V. $\vec{\alpha}$ is referred to as root vector. The set of all root vectors is called root system Σ. The set Σ by itself is not a vector space, but Σ spans V. Furthermore one can define a scalar product $(\,\cdot\,,\,\cdot\,)$ on Σ via

$$(\alpha, \beta) := g^{ij} \alpha_i \beta_j \qquad \text{(B.22)}$$

where the metric tensor g^{ij} is given by

$$g^{ij} = g_{ij} := \sum_\alpha \alpha_i \alpha_j \ . \qquad \text{(B.23)}$$

The length of a root vector is then defined by $|\alpha| := \sqrt{(\alpha, \alpha)}$. A semi-simple Lie group G can now be uniquely characterized by the root system Σ of the corresponding Lie algebra $\mathfrak{g}$ [57, 56, 58], where the roots are normalized such that the shortest root has length $\sqrt{2}$.

B.4 Definition of E_6

The E_6 is a compact simple Lie group of rank 6. It is characterized by the root system Σ of the corresponding Lie algebra $\mathfrak{e}_6$

$$\Sigma = \left\{ \vec{\alpha} \in \mathbb{Z}^6 \cup \left(\mathbb{Z} + \frac{1}{2} \right)^6 : \sum_{i=1}^{6} \alpha_i^2 + 2\alpha_1^2 = 2, \sum_{i=1}^{6} \alpha_i + 2\alpha_1 = \text{even} \right\} \ . \qquad \text{(B.24)}$$

where $|\Sigma| = 72$. Together with the 6 null vectors, $\mathfrak{e}_6$ has 78 generators. The roots are normalized such that the shortest root has length $\sqrt{2}$.

B.5 E_6 representation decomposition

Representations of groups can in general be decomposed in terms of tensor products of representations of subgroups. The decompositions for groups used in this text are listed here.

The decomposition of the representation of E_6 under $SO(10) \times U(1)_\psi$ reads

$$(\mathbf{27})_{E_6} \to (\mathbf{16}, 1) + (\mathbf{10}, -2) + (\mathbf{1}, 4) \qquad \text{(fundamental repr.)} \tag{B.25}$$
$$(\mathbf{78})_{E_6} \to (\mathbf{45}, 0) + (\mathbf{16}, -3) + (\overline{\mathbf{16}}, 3) + (\mathbf{1}, 0) \qquad \text{(adjoint repr.)} \tag{B.26}$$

and the $SO(10)$ representations can again be decomposed under $SU(5) \times U(1)_\chi$ into

$$(\mathbf{10})_{SO(10)} \to (\mathbf{5}, -2) + (\bar{\mathbf{5}}, 2) \qquad \text{(fundamental repr.)} \tag{B.27}$$
$$(\mathbf{45})_{SO(10)} \to (\mathbf{24}, 0) + (\mathbf{10}, -4) + (\overline{\mathbf{10}}, 4) + (\mathbf{1}, 0) \qquad \text{(adjoint repr.)} \tag{B.28}$$
$$(\mathbf{16})_{SO(10)} \to (\mathbf{10}, 1) + (\bar{\mathbf{5}}, -3) + (\mathbf{1}, 5)\ . \tag{B.29}$$

The $SU(5)$ representations decompose under $SU(3)_\mathrm{c} \times SU(2)_\mathrm{L} \times U(1)_Y$

$$(\mathbf{5})_{SU(5)} \to (\mathbf{3}, \mathbf{1}, -\tfrac{1}{3}) + (\mathbf{1}, \mathbf{2}, \tfrac{1}{2}) \qquad \text{(fundamental repr.)} \tag{B.30}$$
$$(\mathbf{24})_{SU(5)} \to (\mathbf{8}, \mathbf{1}, 0) + (\bar{\mathbf{3}}, \mathbf{2}, -\tfrac{5}{6}) + (\mathbf{3}, \mathbf{2}, \tfrac{5}{6}) \tag{B.31}$$
$$+ (\mathbf{1}, \mathbf{3}, 0) + (\mathbf{1}, \mathbf{1}, 0) \qquad \text{(adjoint repr.)} \tag{B.32}$$
$$(\mathbf{10})_{SU(5)} \to (\mathbf{3}, \mathbf{2}, \tfrac{1}{6}) + (\bar{\mathbf{3}}, \mathbf{1}, -\tfrac{2}{3}) + (\mathbf{1}, \mathbf{1}, 1)\ . \tag{B.33}$$

B.6 E_6 covariant derivative

The E_6 covariant derivative reads

$$D_\mu = \partial_\mu + \mathrm{i} g_0 \tilde{A}^{\bar{a}}_\mu \tilde{T}^{\bar{a}}\ , \qquad (\bar{a} = 1, \ldots, 78)\ , \tag{B.34}$$

where $\tilde{T}^{\bar{a}}$ are the generators and $\tilde{A}^{\bar{a}}_\mu$ are gauge fields in the adjoint representation ($\mathbf{78}$) of E_6. The covariant derivative can be decomposed into the interactions of $SU(3)_\mathrm{c}$, $SU(2)_\mathrm{L}$, $U(1)_Y$ and $U(1)_N$[‡]

$$D_\mu = \partial_\mu + \mathrm{i} g_3 T^a A^a_\mu + \mathrm{i} g_2 \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu + \mathrm{i} g_1 \frac{Y}{2} B_\mu + \mathrm{i} g_N \frac{N}{2} Z'_\mu + \cdots \tag{B.36}$$

All generators in (B.36) are normalized such that the quantum numbers are the same as in Table 3.1. This yields a relation between the couplings

$$g_0 = g_3 = g_2 = \sqrt{\frac{5}{3}}\, g_1 = \sqrt{40}\, g_N \tag{B.37}$$

[‡]The notation for the charges in ref. [24] and [1] is slightly different. In order to change to this notation, replace in [24] and [1]

$$\sqrt{\frac{5}{3}}\, Q^Y \to \frac{Y}{2}\ , \qquad \sqrt{40}\, Q^N \to \frac{N}{2}\ , \qquad \sqrt{\frac{3}{5}}\, g_Y \to g_1\ , \qquad \frac{1}{\sqrt{40}}\, g_N \to g_N \tag{B.35}$$

and the relation

$$Q = \frac{\tau_3}{2} + \frac{Y}{2} \tag{B.38}$$

still holds for the so normalized generators.

B.7 E$_6$SSM Lagrangian

The supersymmetric part of the E$_6$SSM Lagrangian can be split into a pure gauge part, which contains only gauge bosons (and their superpartners) and a matter part, which contains all fields and their interactions with gauge fields

$$\mathcal{L}_{\text{SUSY}}^{\text{E}_6\text{SSM}} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{matter}} \,. \tag{B.39}$$

The pure gauge part reads

$$\mathcal{L}_{\text{gauge}} = \frac{1}{4} \int \mathrm{d}^2\theta \left(W_g^{aA} W_{gA}^a + \vec{W}_W^A \cdot \vec{W}_{WA} + W_Y^A W_{YA} + W_N^A W_{NA} \right) + \text{h.c.} \tag{B.40}$$

and the matter part is given by

$$\begin{aligned}
\mathcal{L}_{\text{matter}} = \int \mathrm{d}^4\theta \Big[& L_i^\dagger \exp\left(g_2 \vec{V}^W \cdot \vec{\tau} + g_1 V^Y Y + g_N V^N N \right) L_i \\
& + \bar{E}_i^\dagger \exp\left(g_1 V^Y Y + g_N V^N N \right) \bar{E}_i \\
& + \bar{N}_i^\dagger \bar{N}_i \\
& + Q_i^\dagger \exp\left(g_3 V_g^a \lambda^a + g_2 \vec{V}^W \cdot \vec{\tau} + g_1 V^Y Y + g_N V^N N \right) Q_i \\
& + \bar{U}_i^\dagger \exp\left(-g_3 V_g^a \lambda^a + g_1 V^Y Y + g_N V^N N \right) \bar{U}_i \\
& + \bar{D}_i^\dagger \exp\left(-g_3 V_g^a \lambda^a + g_1 V^Y Y + g_N V^N N \right) \bar{D}_i \\
& + X_i^\dagger \exp\left(g_3 V_g^a \lambda^a + g_1 V^Y Y + g_N V^N N \right) X_i \\
& + \bar{X}_i^\dagger \exp\left(-g_3 V_g^a \lambda^a + g_1 V^Y Y + g_N V^N N \right) \bar{X}_i \\
& + \sum_{p=1}^{2} H_{pi}^\dagger \exp\left(g_2 \vec{V}^W \cdot \vec{\tau} + g_1 V^Y Y + g_N V^N N \right) H_{pi} \\
& + S_i^\dagger \exp\left(g_N V^N N \right) S_i \\
& + H'^\dagger \exp\left(g_2 \vec{V}^W \cdot \vec{\tau} + g_1 V^Y Y + g_N V^N N \right) H' \\
& + \bar{H}'^\dagger \exp\left(g_2 \vec{V}^W \cdot \vec{\tau} + g_1 V^Y Y + g_N V^N N \right) \bar{H}' \\
& + \mathcal{W}_{\text{E}_6\text{SSM}} \delta^{(2)}(\bar{\theta}) + \mathcal{W}_{\text{E}_6\text{SSM}}^\dagger \delta^{(2)}(\theta) \Big] \,,
\end{aligned} \tag{B.41}$$

where $\mathcal{W}_{\text{E}_6\text{SSM}}$ is the E$_6$SSM superpotential (see Eq. (3.47) and (3.51)).

B.8 Higgs potential

The E_6SSM scalar potential is given by the sum of the squared F and D terms plus the soft supersymmetry breaking part

$$V = F_i^* F_i + \frac{1}{2}\left[D^a D^a + \vec{D}^2 + (D^Y)^2 + (D^N)^2\right] + V_{\text{soft}} \ . \tag{B.42}$$

The F and D terms for a specific gauge group with generators T^a and gauge coupling g are given by

$$F_i = -\left.\frac{\partial \mathcal{W}^\dagger_{\text{E}_6\text{SSM}}}{\partial \Phi_i^\dagger}\right|_{\theta=\bar{\theta}=0} \ , \qquad D^a = -g \phi_i^\dagger T^a_{ij} \phi_j \ , \tag{B.43}$$

where the index i runs in gauge representation and generation space. The part of V, which contains only the three Higgs fields H_{13}, H_{23} and S_3 then reads for the E_6SSM

$$V_{\text{Higgs}} = V_F + V_D + V_{\text{soft}} \tag{B.44}$$

$$V_F = |\lambda_3|^2 |s_3|^2 \left(|h_{13}|^2 + |h_{23}|^2\right) + |\lambda_3|^2 |h_{13} \cdot h_{23}|^2 \tag{B.45}$$

$$\begin{aligned} V_D = &\frac{1}{8}(g_1^2 + g_2^2)\left(|h_{13}|^2 - |h_{23}|^2\right)^2 + \frac{g_2^2}{2}|h_{13}^\dagger h_{23}|^2 \\ &+ \frac{g_N^2}{2}\left[\frac{N_{H_{13}}}{2}|h_{13}|^2 + \frac{N_{H_{23}}}{2}|h_{23}|^2 + \frac{N_{S_3}}{2}|s_3|^2\right]^2 \end{aligned} \tag{B.46}$$

$$V_{\text{soft}} = m_{h_{13}}^2 |h_{13}|^2 + m_{h_{23}}^2 |h_{23}|^2 + m_{s_3}^2 |s_3|^2 + \left[\lambda_3 A_{\lambda_3} s_3 (h_{13} \cdot h_{23}) + \text{h. c.}\right] \ , \tag{B.47}$$

where the following notation was used

$$h_{13} \cdot h_{23} \equiv \epsilon_{AB} h_{13}^A h_{23}^B \tag{B.48}$$

$$|h_{13}|^2 \equiv h_{13}^\dagger h_{13} \equiv \sum_{A=1}^{2} (h_{13}^A)^* h_{13}^A \tag{B.49}$$

$$|h_{13}^\dagger h_{23}|^2 = (h_{13}^\dagger h_{23})(h_{23}^\dagger h_{13}) \ . \tag{B.50}$$

The quantum numbers $N/2$ for the Higgs fields are listed in Table 3.1. The Higgs fields get non-zero VEVs

$$\langle h_{13} \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_1 \\ 0 \end{pmatrix} \ , \qquad \langle h_{23} \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix} \ , \qquad \langle s_3 \rangle = \frac{s}{\sqrt{2}} \tag{B.51}$$

at the minima of V_{Higgs}, which are given by the three electroweak symmetry breaking conditions

$$\begin{aligned}0 = \frac{\partial V_{\text{Higgs}}}{\partial s} &= m_{s_3}^2 s - \frac{\lambda_3 A_{\lambda_3}}{\sqrt{2}} v_1 v_2 + \frac{\lambda_3^2}{2}(v_1^2 + v_2^2) s \\ &+ \frac{g_N^2}{2}\left(\frac{N_{H_{13}}}{2} v_1^2 + \frac{N_{H_{23}}}{2} v_2^2 + \frac{N_{S_3}}{2} s^2\right) \frac{N_{S_3}}{2} s\end{aligned} \tag{B.52}$$

$$\begin{aligned}0 = \frac{\partial V_{\text{Higgs}}}{\partial v_1} &= m_{h_{13}}^2 v_1 - \frac{\lambda_3 A_{\lambda_3}}{\sqrt{2}} s v_2 + \frac{\lambda_3^2}{2}(v_2^2 + s^2) v_1 + \frac{\bar{g}^2}{8}(v_1^2 - v_2^2) v_1 \\ &+ \frac{g_N^2}{2}\left(\frac{N_{H_{13}}}{2} v_1^2 + \frac{N_{H_{23}}}{2} v_2^2 + \frac{N_{S_3}}{2} s^2\right) \frac{N_{H_{13}}}{2} v_1\end{aligned} \tag{B.53}$$

$$\begin{aligned}0 = \frac{\partial V_{\text{Higgs}}}{\partial v_2} &= m_{h_{23}}^2 v_2 - \frac{\lambda_3 A_{\lambda_3}}{\sqrt{2}} s v_1 + \frac{\lambda_3^2}{2}(v_1^2 + s^2) v_2 + \frac{\bar{g}^2}{8}(v_2^2 - v_1^2) v_2 \\ &+ \frac{g_N^2}{2}\left(\frac{N_{H_{13}}}{2} v_1^2 + \frac{N_{H_{23}}}{2} v_2^2 + \frac{N_{S_3}}{2} s^2\right) \frac{N_{H_{23}}}{2} v_2 \ ,\end{aligned} \tag{B.54}$$

where $\bar{g}^2 = g_1^2 + g_2^2$.

B.9 Renormalization group equations

The one-loop beta function for a supersymmetric gauge theory with gauge coupling g is given by

$$\beta = -3C_v + \sum_i C_i \ , \tag{B.55}$$

where it is summed over all chiral superfields Φ_i which couple to g. The corresponding renormalization group equation reads

$$\frac{\mathrm{d}g}{\mathrm{d}t} = \frac{\beta g^3}{(4\pi)^2} \ , \qquad t = \log \mu \tag{B.56}$$

and the used group factors are defined by

$$C_i = \begin{cases} \frac{1}{2} & \text{for } SU(N) \\ \left(\frac{Y_i}{2}\right)^2 & \text{for } U(1)_Y \\ \left(\frac{N_i}{2}\right)^2 & \text{for } U(1)_N \end{cases} \quad \text{and} \quad C_v = \begin{cases} N & \text{for } SU(N) \\ 0 & \text{for } U(1) \end{cases} \tag{B.57}$$

In the $\mathrm{E_6SSM}$ one has in particular

$$\beta_3^{\mathrm{E_6SSM}} = -9 + 3N_g \qquad \text{for} \qquad SU(3)_{\mathrm{c}} \tag{B.58}$$

$$\beta_2^{\mathrm{E_6SSM}} = -5 + 3N_g \qquad \text{for} \qquad SU(2)_{\mathrm{L}} \tag{B.59}$$

$$\beta_1^{\mathrm{E_6SSM}} = 1 + 5N_g \qquad \text{for} \qquad U(1)_Y \tag{B.60}$$

$$\beta_N^{\mathrm{E_6SSM}} = 16 + 120 N_g \qquad \text{for} \qquad U(1)_N\ , \tag{B.61}$$

where $N_g = 3$ is the number of generations. The quantum numbers $N_i/2$ and $Y_i/2$ for the chiral superfields of the E$_6$SSM are listed in Table 3.1. Note that in the E$_6$SSM the beta function for the strong coupling vanishes at one-loop level, $\beta_3^{\mathrm{E_6SSM}} = 0$. This implies at one-loop that the $SU(3)_\mathrm{c}$ is not asymptotically free. At two-loop level one even has $\beta_3^{(2\ \mathrm{loop}),\mathrm{E_6SSM}} > 0$ for all parameter points studied in [1]. For completeness the Standard Model beta functions are listed

$$\beta_3^{\mathrm{SM}} = -7 \qquad \text{for} \qquad SU(3)_\mathrm{c} \tag{B.62}$$

$$\beta_2^{\mathrm{SM}} = -\frac{19}{6} \qquad \text{for} \qquad SU(2)_\mathrm{L} \tag{B.63}$$

$$\beta_1^{\mathrm{SM}} = \frac{41}{6} \qquad \text{for} \qquad U(1)_Y\ . \tag{B.64}$$

B.10 Feynman rules for the exotic quarks

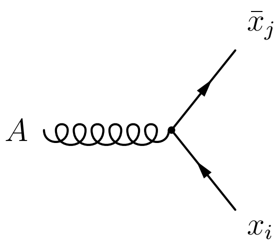
Here the Feynman rules for the exotic quarks are listed, which were used in the calculations in Chapter 4. They follow from the Lagrangian

$$\mathcal{L} = \sum_{i=1}^{3}\left(\bar{x}_i \mathrm{i}\not{D} x_i + |D_\mu \tilde{x}_{iL}|^2 + |D_\mu \tilde{x}_{iR}|^2\right) = \sum_{i=1}^{3} \bar{x}_i \mathrm{i}\not{D} x_i + \sum_{i=1}^{3}\sum_{k=1}^{2} |D_\mu \tilde{x}_{ik}|^2\ , \tag{B.65}$$

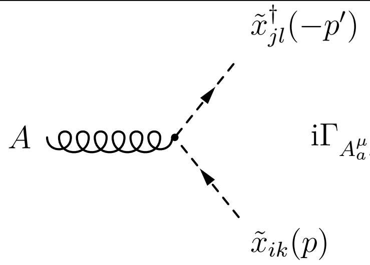
where i is the generation index and k labels the mass eigenstates. In the last step of Eq. (B.65) the gauge eigenstates $\tilde{x}_{iL}$ and $\tilde{x}_{iR}$ were unitarily transformed into mass eigenstates

$$\begin{pmatrix}\tilde{x}_{iL}\\ \tilde{x}_{iR}\end{pmatrix} = W^{\tilde{x}_i}\begin{pmatrix}\tilde{x}_{i1}\\ \tilde{x}_{i2}\end{pmatrix} = \begin{pmatrix}\cos\theta_{\tilde{x}_i} & -\sin\theta_{\tilde{x}_i}\\ \sin\theta_{\tilde{x}_i} & \cos\theta_{\tilde{x}_i}\end{pmatrix}\begin{pmatrix}\tilde{x}_{i1}\\ \tilde{x}_{i2}\end{pmatrix}\ . \tag{B.66}$$

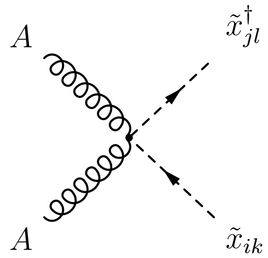
For the derivation of the Feynman rules the covariant derivative in Eq. (B.36) in the Haber–Kane convention is used and it is assumed for simplicity that the Z' does not mix with the other gauge bosons.



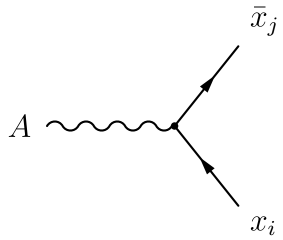
$$\mathrm{i}\Gamma_{A_a^\mu x_i \bar{x}_j} = -\mathrm{i} g_3 \gamma_\mu T^a \delta_{ij} \tag{B.67}$$



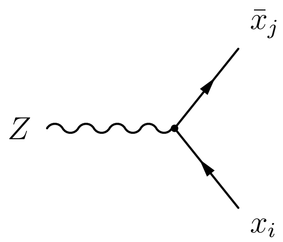
$$\mathrm{i}\Gamma_{A^\mu_a \tilde{x}_{ik}\tilde{x}^\dagger_{jl}}(k,p,-p') = -\mathrm{i}g_3(p+p')_\mu T^a \delta_{ij}\delta_{kl} \tag{B.68}$$



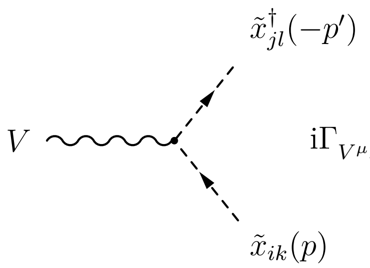
$$\mathrm{i}\Gamma_{A^\mu_a A^\nu_b \tilde{x}_{ik}\tilde{x}^\dagger_{jl}} = \mathrm{i}g_3^2\, g_{\mu\nu}\{T^a, T^b\}\delta_{ij}\delta_{kl} \tag{B.69}$$



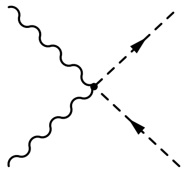
$$\mathrm{i}\Gamma_{A^\mu x_i \bar{x}_j} = -\mathrm{i}eQ\gamma_\mu\delta_{ij}\ , \qquad Q = \frac{Y}{2} = -\frac{1}{3} \tag{B.70}$$



$$\mathrm{i}\Gamma_{Z^\mu x_i \bar{x}_j} = \mathrm{i}eQ\frac{s_W}{c_W}\gamma_\mu\delta_{ij}\ , \qquad Q = \frac{Y}{2} = -\frac{1}{3} \tag{B.71}$$



$$\mathrm{i}\Gamma_{V^\mu \tilde{x}_{ik}\tilde{x}^\dagger_{jl}} = \mathrm{i}eQ(p+p')_\mu\delta_{ij}\delta_{kl}\begin{cases} -1 & \text{for} \quad V^\mu \equiv A^\mu \\ \frac{s_W}{c_W} & \text{for} \quad V^\mu \equiv Z^\mu \end{cases} \tag{B.72}$$



$$
\mathrm{i}\Gamma_{V^\mu V'^\nu \tilde{x}_{ik}\tilde{x}^\dagger_{jl}} = 2\mathrm{i}e^2 Q^2 g_{\mu\nu}\delta_{ij}\delta_{kl}
$$

$$
\times \begin{cases} 1 & \text{for} \quad V^\mu \equiv A^\mu, V'^\mu \equiv A^\mu \\ -\frac{s_W}{c_W} & \text{for} \quad V^\mu \equiv A^\mu, V'^\mu \equiv Z^\mu \\ \left(\frac{s_W}{c_W}\right)^2 & \text{for} \quad V^\mu \equiv Z^\mu, V'^\mu \equiv Z^\mu \end{cases} \tag{B.73}
$$

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Alexander Voigt
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