Photonic two-loop corrections to the muon's anomalous magnetic moment $(g-2)_\mu$ in the MSSM

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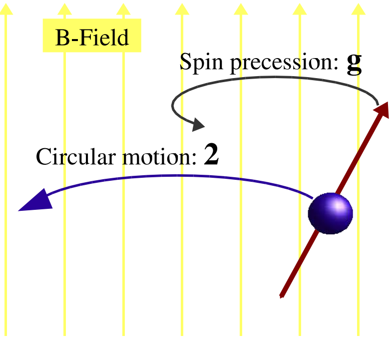
9 July 2009

$3.4\,\sigma$

What's $g - 2$?



$$H_{\text{magn}} = -{\color{red}g}\frac{e}{2m}\vec{B}\vec{S}$$

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Simple to measure

e.g. $(g-2)_e$ used to measure α_{QED}

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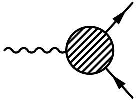
Sensitive to quantum corrections (birth of QFT)

One of the most precisely measured and computed quantities

Computing $g - 2$ in QFT



$$\sim -ie\,\bar{u}(p')\left[\gamma^\mu F_E(q^2) + (p+p')^\mu F_M(q^2) + \ldots\right] u(p)$$

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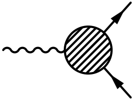

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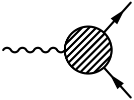

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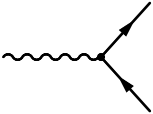

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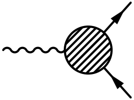

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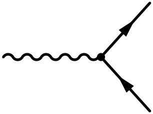

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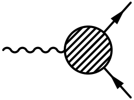 $\sim -ie\,\bar{u}(p')\left[\gamma^\mu F_E(q^2) + (p+p')^\mu F_M(q^2) + \ldots\right] u(p)$

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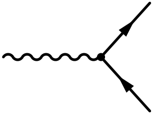
$\mathbf{g-2=0}$ on tree level (cf. Dirac eq)  $\sim -ie\gamma^\mu$

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Loops with heavy particles (mass M):
$g-2 \propto \frac{m^2}{M^2} \Rightarrow$ muon ca. **40,000** times more sensitive

History: experiment and SM predictions

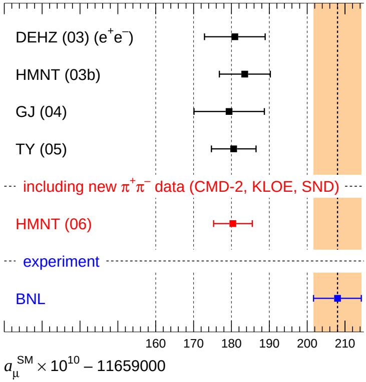
$\pm$	a_μ	σ_{a_μ}/a_μ	Exp./SM
	$\alpha/2\pi$		'49 Schwinger: QED 1L
	-1.77×10^{-6}		'50-'57 QED 2L terms
μ^+	± 0.10		'57 Nevis
μ^+	$0.001\,13^{+0.000\,16}_{-0.000\,12}$	12.4%	'59 Nevis
μ^+	$0.001\,145(22)$	1.9%	'61 CERN 1(SC)
$\mu^\pm$	$0.001\,166\,16(31)$	265ppm	'68 CERN 2(PS)
	$+5.9 \times 10^{-6}$		'72 QED 2L $\log \frac{m_\mu}{m_e}$ terms
$\mu^\pm$	$0.001\,165\,895(27)$	23ppm	'75 CERN 3(PS)
	$+3 \times 10^{-7}$		'76 QED 3L numerically
	$(4 \pm 2) \times 10^{-9}$		'76 QED 4L leading
	$(6.7 \pm .9) \times 10^{-8}$		'76 hadronic contributions
$\mu^\pm$	$0.001\,165\,911(11)$	7.3ppm	'79 CERN 3(PS)

$(g-2)_\mu$ experiment in Brookhaven

$$a_\mu^{Exp} = 11\,659\,208.0(6.3)\times 10^{-10}$$

[Phys.Rev.D 2006]

Experiment vs. SM predictions



$3.4\,\sigma$ established

$$(29.5 \pm 8.1) \times 10^{-10}$$

[de Rafael 2008]

Scenarios

Higher statistical error in experiment

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Missing higher orders of SM contributions

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Underestimated theory error

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New physics!

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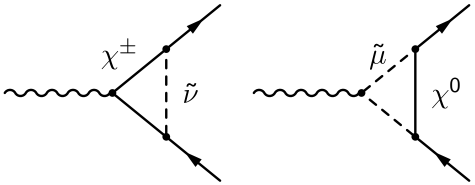
New physics! Supersymmetry (MSSM)

$(g-2)_\mu$ in the MSSM

SUSY 1 loop



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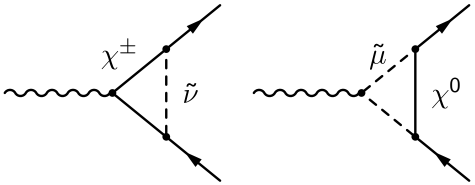

$$\propto m_\mu \cdot \tan\beta \cdot \mu$$

complete

[Fayet '80]
[Kosower et al '83]
[Lopez et al '94]

[Moroi '96]

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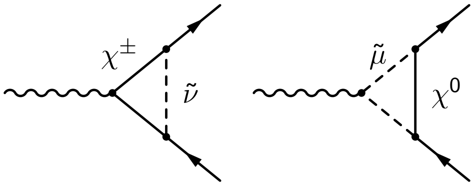


for $\mu,\ m_{\tilde{\mu}},\ m_{\tilde{\chi}} \approx M_{\mathrm{SUSY}}$

$$a_\mu^{\mathrm{SUSY}} \approx 12 \times 10^{-10}\ \tan\beta\ \mathrm{sgn}(\mu) \left(\frac{100\ \mathrm{GeV}}{M_{\mathrm{SUSY}}}\right)^2$$

e.g. $a_\mu^{\mathrm{SUSY}} = 24 \times 10^{-10}$ for

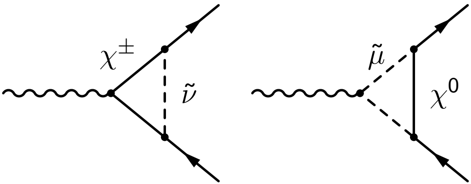
$\tan\beta = 2, \quad M_{\mathrm{SUSY}} = 100\ \mathrm{GeV}$

$\tan\beta = 50, \quad M_{\mathrm{SUSY}} = 500\ \mathrm{GeV}$

SUSY could account for $3.4\,\sigma$

$g - 2$ constraints MSSM parameters

SUSY 1 loop



$$a_\mu^{\chi^\pm} = \frac{m_\mu}{16\pi^2}\sum_k \left[\frac{m_\mu}{12 m_{\tilde\nu_\mu}^2}\left(|c_k^L|^2 + |c_k^R|^2\right)F_1^C(x) + \frac{2m_{\chi^\pm}}{3m_{\tilde\nu_\mu}^2}\mathrm{Re}[c_k^L c_k^R]F_2^C(x)\right]$$

$$a_\mu^{\chi^0} = \frac{m_\mu}{16\pi^2}\sum_{i,m} \left[-\frac{m_\mu}{12 m_{\tilde\mu}^2}\left(|n_{im}^L|^2 + |n_{im}^R|^2\right)F_1^N(x) + \frac{m_{\chi^0}}{3m_{\tilde\mu}^2}\mathrm{Re}[n_{im}^L n_{im}^R]F_2^N(x)\right]$$

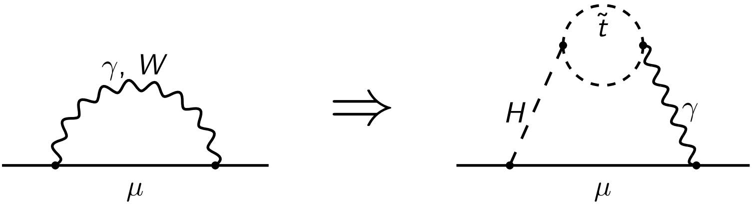
$$F_1^C = \frac{2}{(1-x)^4}\left(2 + 3x - 6x^2 + x^3 + 6x\log x\right)$$

$$F_2^C = \frac{3}{2(1-x)^3}\left(-3 + 4x - x^2 - 2\log x\right)$$

$$F_1^N = \frac{2}{(1-x)^4}\left(1 - 6x + 3x^2 + 2x^3 - 6x^2\log x\right)$$

$$F_2^N = \frac{3}{(1-x)^3}\left(1 - x^2 + 2x\log x\right)$$

2 loop: SUSY-enhanced SM loop



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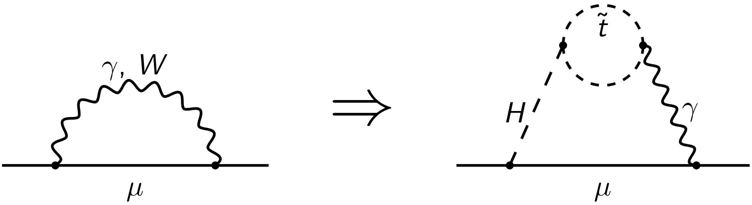
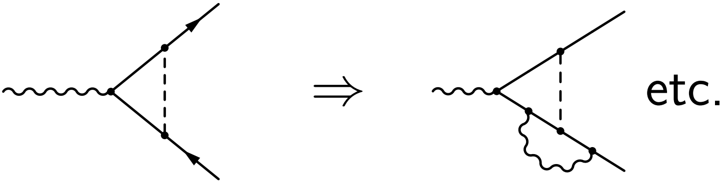


$$\propto m_t \cdot \tan\beta \cdot \mu$$

complete

[Chen, Geng '01]
[Arhib, Baek '02]
[Stöckinger et al '03]
[Stöckinger et al '04]

2 loop: Photon enhanced SUSY loop



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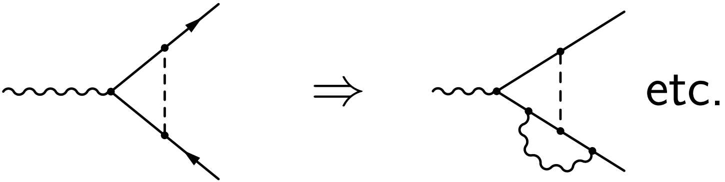

$$\propto \log \frac{M_{\mathrm{SUSY}}}{m_\mu} \; a_\mu^{1\mathrm{L}}$$

leading log

[Degrassi, Giudice '98]

goal: complete

Methods of computation

Simplified model

$$\text{QED} + \chi^{\pm} + \chi^{0} + \tilde{\mu} + \tilde{\nu}$$

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Chargino and neutralino coupling arbitrary for now:

$$\begin{aligned}\mathscr{L}_{\mathrm{int}} &= \overline{\mu}(n_L P_R - {n_R}^* P_L)\chi^0\tilde{\mu} \\ &\quad + \overline{\mu}(c_L P_R + {c_R}^* P_L)\chi^-\tilde{\nu} + \text{h.c.}\end{aligned}$$

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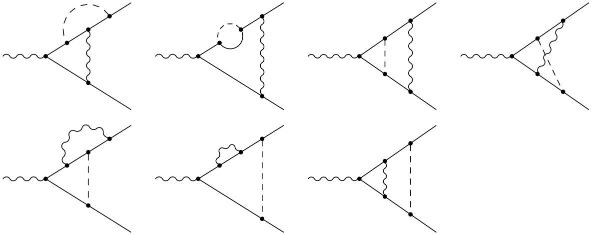
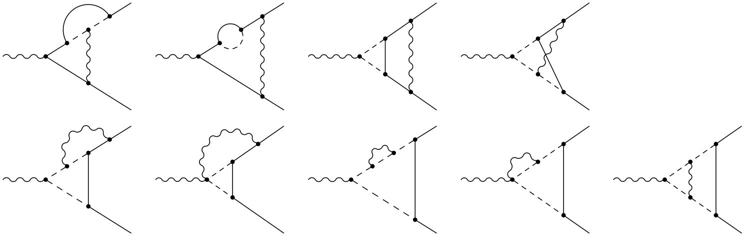
Later:

Plug in values from gauge and Yukawa couplings.

Sum over gauginos and muons in the MSSM.

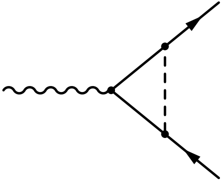
FeynArts finds 28 unique diagrams $\propto c^2e^2, n^2e^2$





FeynArts applies Feynman rules

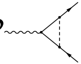
For example:



$$= \int \frac{i(c_R^* P_L + c_L P_R)(\not{k} + \not{q} + m_\chi)(-ie\gamma^\mu)(\not{k} + m_\chi) i(c_L P_L + c_R P_R)}{[k^2 - m_\chi^2][(k+q)^2 - m_\chi^2][(k - p' + q)^2 - m_{\tilde{\nu}}^2]}$$

Extract $g-2$ from diagram amplitude

Idea: use **projector** $\mathcal{P}$ to extract $g-2$ from amplitude:

$$a_\mu = \mathrm{Tr}\Big(\mathcal{P}$$

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$$= c_\mu V^\mu + c_{\mu\nu} T^{\mu\nu}$$

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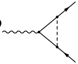
$a_\mu = \mathrm{Tr}\Big(\mathcal{P}$  $\Big)$

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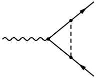
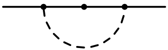
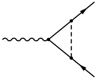
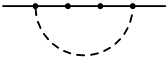
$V^\mu \sim$  $\Big|_{q^2=0} \sim$ 

$T^{\mu\nu} \sim \dfrac{\partial}{\partial q^\nu}$  $\Big|_{q^2=0} \sim$ 

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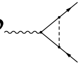
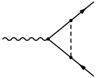


$$\Big) = c_\mu V^\mu + c_{\mu\nu} T^{\mu\nu}$$

$$V^\mu \sim$$



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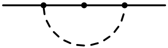

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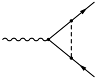

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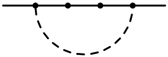

V^μ, $T^{\mu\nu}$ look like **selfenergies**!

TwoCalc/OneCalc evaluates selfenergies

Perform Dirac calculus
e.g. $\gamma_\nu \gamma_\mu \gamma^\nu = (2 - D)\gamma_\mu$

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Express result in standard integral notation:

$$\int \frac{\cdots}{[k_1^2 - m_1^2][k_2^2 - m_2^2][\ldots]} \longrightarrow A_0, B_0, T_{123}, Y_{1234}^{12}, \text{ etc.}$$

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Goal: Decompose to **tabulated integrals**

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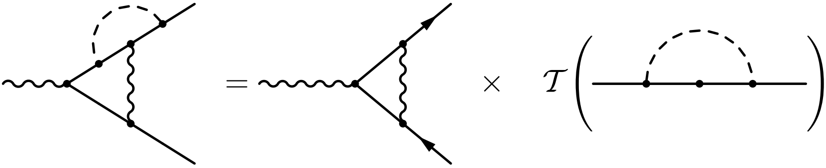
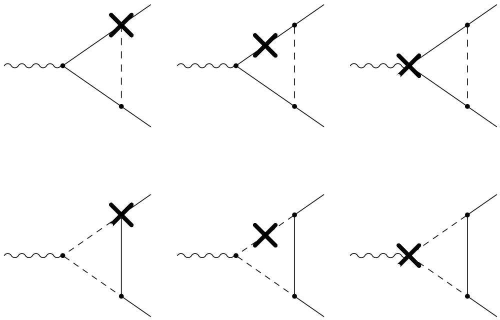
Solution: only expand loops with **large masses**



$$+\,\mathcal{O}(m_\mu^3/M_{\mathrm{SUSY}}^3)$$

Renormalization: 10 unique counterterm diagrams



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$\chi^\pm$ and $\tilde{\mu}$ masses **on-shell** (not possible for both smuons in the MSSM)

Idea: couplings c, n in **$\overline{\text{MS}}$ scheme**

Problem: c, n contain m_μ (**large logarithms**), need to account for that:

$$\delta(c_L c_R) = \frac{\delta m_\mu}{m_\mu} c_L c_R + \dots$$
$$\delta(n_L n_R) = \frac{\delta m_\mu}{m_\mu} n_L n_R + \mathcal{O}(m_\mu^2) + \dots$$

Analytical result

$$a_{2\mathrm{L}}^{\chi^\pm} = \frac{1}{16\pi^2}\frac{e^2}{2\pi^2}\Bigg[\left(\frac{m_\mu^2}{12m_{\tilde\nu}^2}F_1^C\mathscr{A}_C + \frac{2m_\mu m_{\chi^\pm}}{3m_{\tilde\nu}^2}F_2^C\mathscr{R}_C\right)\log\frac{m_\mu^2}{m_{\tilde\nu}^2}$$
$$-\left(\frac{47m_\mu^2}{576m_{\tilde\nu}^2}F_3^C\mathscr{A}_C + \frac{61m_\mu m_{\chi^\pm}}{36m_{\tilde\nu}^2}F_4^C\mathscr{R}_C\right)$$
$$-\left(\frac{m_\mu^2}{16m_{\tilde\nu_\mu}^2}F_1^C\mathscr{A}_C + \frac{m_\mu m_{\chi^\pm}}{4m_{\tilde\nu}^2}F_2^C\mathscr{R}_C\right)\mathrm{L}(m_{\tilde\nu}^2)\Bigg]$$

$$a_{2\mathrm{L}}^{\chi^0} = \frac{1}{16\pi^2}\frac{e^2}{2\pi^2}\Bigg[\left(-\frac{m_\mu^2}{12m_{\tilde\mu}^2}F_1^N\mathscr{A}_N + \frac{m_\mu m_{\chi^0}}{3m_{\tilde\mu}^2}F_2^N\mathscr{R}_N\right)\log\frac{m_\mu^2}{m_{\tilde\mu}^2}$$
$$-\left(-\frac{35m_\mu^2}{576m_{\tilde\mu}^2}F_3^N\mathscr{A}_N + \frac{2m_\mu m_{\chi^0}}{9m_{\tilde\mu}^2}F_4^N\mathscr{R}_N\right)$$
$$-\left(\frac{m_\mu^2}{32m_{\tilde\mu}^2}F_1^N\mathscr{A}_N\right)\mathrm{L}(m_{\tilde\mu}^2)\Bigg]$$

Analytical result

$$
\begin{aligned}
a_{2\mathrm{L}}^{\chi^\pm} = \frac{1}{16\pi^2}\frac{e^2}{2\pi^2}\Bigg[& \left(\frac{m_\mu^2}{12 m_{\tilde\nu}^2} F_1^C \mathscr{A}_C + \frac{2 m_\mu m_{\chi^\pm}}{3 m_{\tilde\nu}^2} F_2^C \mathscr{R}_C\right) \log\frac{m_\mu^2}{m_{\tilde\nu}^2} \\
& - \left(\frac{47 m_\mu^2}{576 m_{\tilde\nu}^2} F_3^C \mathscr{A}_C + \frac{61 m_\mu m_{\chi^\pm}}{36 m_{\tilde\nu}^2} F_4^C \mathscr{R}_C\right) \\
& - \left(\frac{m_\mu^2}{16 m_{\tilde\nu_\mu}^2} F_1^C \mathscr{A}_C + \frac{m_\mu m_{\chi^\pm}}{4 m_{\tilde\nu}^2} F_2^C \mathscr{R}_C\right) \mathrm{L}(m_{\tilde\nu}^2) \Bigg]
\end{aligned}
$$

$$
\begin{aligned}
a_{2\mathrm{L}}^{\chi^0} = \frac{1}{16\pi^2}\frac{e^2}{2\pi^2}\Bigg[& \left(-\frac{m_\mu^2}{12 m_{\tilde\mu}^2} F_1^N \mathscr{A}_N + \frac{m_\mu m_{\chi^0}}{3 m_{\tilde\mu}^2} F_2^N \mathscr{R}_N\right) \log\frac{m_\mu^2}{m_{\tilde\mu}^2} \\
& - \left(-\frac{35 m_\mu^2}{576 m_{\tilde\mu}^2} F_3^N \mathscr{A}_N + \frac{2 m_\mu m_{\chi^0}}{9 m_{\tilde\mu}^2} F_4^N \mathscr{R}_N\right) \\
& - \left(\frac{m_\mu^2}{32 m_{\tilde\mu}^2} F_1^N \mathscr{A}_N\right) \mathrm{L}(m_{\tilde\mu}^2) \Bigg]
\end{aligned}
$$

Analytical result

$$a_{2\mathrm{L}}^{\chi^\pm} = \frac{1}{16\pi^2}\frac{e^2}{2\pi^2}\Bigg[a_{1\mathrm{L}}^{\chi^\pm}\log\frac{m_\mu^2}{m_{\tilde{\nu}}^2} - \left(\frac{47m_\mu^2}{576m_{\tilde{\nu}}^2}F_3^C\mathscr{A}_C + \frac{61m_\mu m_{\chi^\pm}}{36m_{\tilde{\nu}}^2}F_4^C\mathscr{R}_C\right) - \left(\frac{m_\mu^2}{16m_{\tilde{\nu}_\mu}^2}F_1^C\mathscr{A}_C + \frac{m_\mu m_{\chi^\pm}}{4m_{\tilde{\nu}}^2}F_2^C\mathscr{R}_C\right)\mathrm{L}(m_{\tilde{\nu}}^2)\Bigg]$$

$$a_{2\mathrm{L}}^{\chi^0} = \frac{1}{16\pi^2}\frac{e^2}{2\pi^2}\Bigg[a_{1\mathrm{L}}^{\chi^\pm}\log\frac{m_\mu^2}{m_{\tilde{\mu}}^2} - \left(-\frac{35m_\mu^2}{576m_{\tilde{\mu}}^2}F_3^N\mathscr{A}_N + \frac{2m_\mu m_{\chi^0}}{9m_{\tilde{\mu}}^2}F_4^N\mathscr{R}_N\right) - \left(\frac{m_\mu^2}{32m_{\tilde{\mu}}^2}F_1^N\mathscr{A}_N\right)\mathrm{L}(m_{\tilde{\mu}}^2)\Bigg]$$

Conclusion

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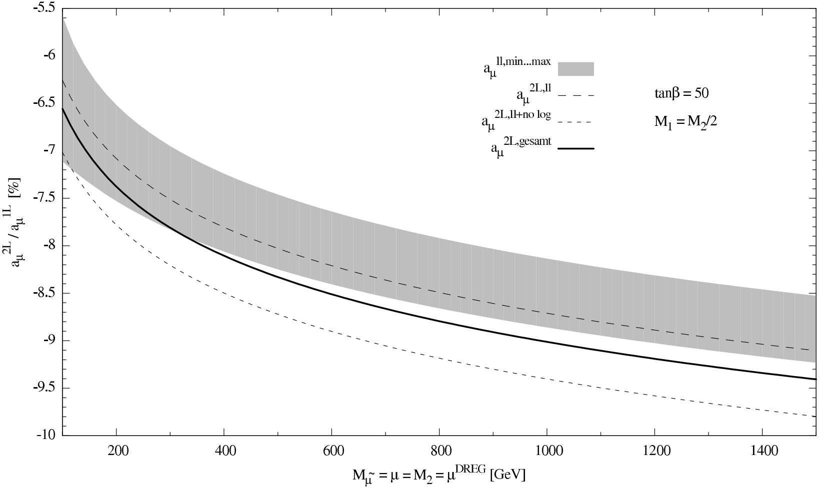
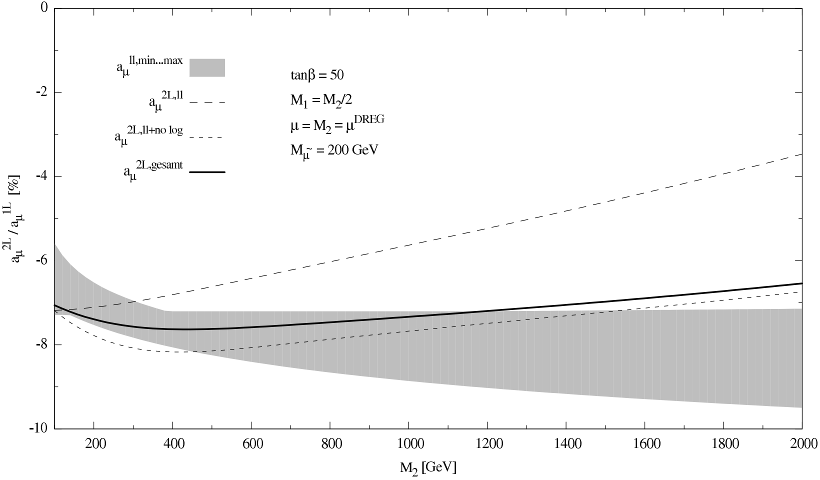
$\surd$ Non-logarithmic contributions complete

Numerical evaluation



Can non-logarithmic contributions be large?



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Non-log contribution amounts to ca. 1% of the one loop result.

Questions?